Chapter notes: 25 Mathematical induction

Overview

This chapter brings together many different areas of the course, so is a useful finishing point. Sections 25B and 25C require knowledge of the notations of sequences and series from sections 7A and 7B. Section 25D requires knowledge of calculus up to the product rule (chapter 18). Sections 25A, 25E and 25F have no prerequisites within the core. It requires approximately four hours of teaching time.

Introductory problem

This problem (called Moser's circle problem) highlights the difference between mathematics and science. It is not sufficient in mathematics to see that a pattern works for the first few cases – or even the first million cases. It is always possible that the pattern will break down. The worked solution is given at the end of the chapter, page 806; the idea being that students should be able to answer the question using the methods covered in the chapter.

25A The principle of mathematical induction, p791

There are no specific teacher notes for this section.

25B Induction and series, p793

Students often attempt each step in an induction proof, but fail to show the flow of logic in their workings. In particular, it is important that they are clear about what has been assumed and what has been derived from these assumptions.

Hints for grade 7 questions:

7. You do not need to consider odd and even k separately – take out a factor of $(-1)^k$ once you have completed the assumption.

8. In the inductive step you will need to add on two terms but also subtract a term.

9. You will need to use a letter other than k for your assumption.

25C Induction and sequences, p795

There have been instances in examination questions where two base cases have been required. Students often find this idea tricky.

Hints for grade 7 questions:

8. This is a slightly unusual recurrence relation, but do not be intimidated. The usual method will work.

9. It is useful to note that for both $\phi = \frac{1+\sqrt{5}}{2}$ and $\phi = \frac{1-\sqrt{5}}{2}$, $\phi^2 = \phi + 1$.

25D Induction and differentiation, p799

Hints for grade 7 questions:

6. You will need to use the Pascal Triangle identity for binomial coefficients:
$\binom{n+1}{k+1} = \binom{n}{k} + \binom{n}{k+1}$.

25E Induction and divisibility, p801

The 'Research explorer 'box on page 802 refers to 'Fermat's little theorem', which states that if p is prime, $a^p \equiv a \bmod p$.

We could use this to show, for example, that $7^{11} + 4^{11}$ is divisible by 11.

Hints for grade 7 questions:

9. In the inductive step add on one cube and subtract another.

25F Induction and inequalities, p803

This topic rarely appears in examinations, so you may wish to omit this section.

Hints for grade 7 questions:

4. Use the fact that $k + 1 > 2$ if $k > 4$.

5. Use the fact that $k(k + 1) > k^2$.

6. Use the fact that $k + \frac{3}{2} = \sqrt{k^2 + 3k + \frac{9}{4}} > \sqrt{k^2 + 3k + 2}$.

7. Use the fact that $k + 1 > 3$ when $k > N$.

8. Use the fact that $x^2 \geq 0$.