

Revision: Statistics and probability (Topic 5)**Coursebook chapters: 21–24**

1. Find the mean and standard deviation of the data given by the following grouped frequency table:

Group	Frequency
6–12	26
12–17	18
17–22	45

(accessible to students on the path to grade 3 or 4) [4 marks]

2. A child looks for seashells lying on the beach. On average, there are 3 shells in every square metre of sand where she is searching. What is the probability that there are more than 20 in a 6 m² area?

(accessible to students on the path to grade 3 or 4) [4 marks]

3. X is a continuous random variable with a pdf:

$$f(x) = \begin{cases} \frac{x-k}{2} & k \leq x \leq k+2 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Given $E(X) = 3$, show that $k = \frac{5}{3}$.

- (b) Find the variance of X .

(accessible to students on the path to grade 3 or 4) [6 marks]

4. John takes the bus to work if it is raining or if he leaves home late; if he is on time and it is not raining, he walks to work. The probability that it rains on a given day is 0.3. The probability that he leaves home late is 0.6, independent of the weather.

- (a) What is the probability that John walks to work?
- (b) Given he took the bus, what is the probability that it was raining?

(accessible to students on the path to grade 3 or 4) [6 marks]

5. Measured salt levels in water samples taken from a lake are seen to follow a normal distribution with mean 0.3 and standard deviation 0.04. Two measurements are taken. Find the probability that:

- (a) both measurements are greater than 0.36
- (b) one measurement is greater than 0.4 and one is less than 0.4.

(accessible to students on the path to grade 3 or 4) [6 marks]

6. X is a continuous random variable with a pdf:

$$f(x) = \begin{cases} 5 - 2x & \text{for } a \leq x \leq 2a \\ 0 & \text{otherwise} \end{cases}$$

Determine the value of a .

(accessible to students on the path to grade 5 or 6) [5 marks]

7. Greig has a coin with a probability p of returning 'Heads' when flipped.

After four coin-tosses, Greig has one 'Tails' and three 'Heads'.

- (a) Find the probability, in terms of p , of flipping three or four 'Heads' out of four coin-tosses.
- (b) What is the least value of p such that the probability of flipping three or four 'Heads' out of four coin-tosses is at least 5%?

(accessible to students on the path to grade 5 or 6) [5 marks]

8. Paul is playing a card game. On each hand he can score 0, 1 or 2 points, with probability $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{6}$ respectively.

- (a) What is the expected score from a single hand?
- (b) At the end of the fourth hand, Paul has scored exactly 3 points. What is the probability that he ever had a 2-point hand?

(accessible to students on the path to grade 5 or 6) [7 marks]

9. A factory produces tins of beans with masses normally distributed with mean 252 g. In order to label the tins as containing a minimum 250 g of product, the manufacturer must ensure that 99% of tins have at least this mass. What standard deviation is acceptable?

(accessible to students on the path to grade 5 or 6) [5 marks]

10. Vesna and Stephen each played a game at a fairground.

Vesna's game involved shooting an air-rifle at moving targets. She can be expected to hit, on average, one target every 40 seconds. If she hit at least five targets in 2 minutes, she won a prize. There was no restriction on the number of shots she could take in the 2 minute period.

Stephen's game involved using a bow to shoot arrows at a target. He had a 0.7 probability of hitting on each shot (independent of previous results) and had 10 arrows. If he could hit the target eight or more times, he won a prize.

Given that exactly one of them won a prize, what is the probability that it was Vesna?

(accessible to students on the path to grade 5 or 6) [7 marks]

11. A particular breed of chicken produces eggs whose masses are known to follow a normal distribution with mean 50 g and standard deviation 4 g.

It is found that the probability of any egg with mass in excess of 60 g being 'double-yolked' is 10%.

Find the probability of seeing exactly 1 double-yolked egg in a random sample of 12 eggs.

(accessible to students on the path to grade 5 or 6) [5 marks]

12. X is a continuous random variable with a pdf:

$$f(x) = k(|x^2 - a^2| - (x^2 - a^2))$$

- (a) Sketch the graph of $f(x)$.
- (b) Write down the range of possible values for X .
- (c) Find an equation for k in terms of a .

(accessible to students on the path to grade 7) [8 marks]

13. X is a discrete random variable. For a given function g , expectation $E(g(X))$ is given by:

$$E(g(X)) = \sum_x P(X=x)g(x)$$

Show that if $X \sim B(n, p)$ then $E(a^X) = (1 - p + pa)^n$.

(accessible to students on the path to grade 7) [5 marks]

14. X is a continuous random variable with a pdf:

$$f(x) = \begin{cases} \frac{2-x}{x+1} & a \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$$

- (a) Sketch the graph of $y = \frac{2-x}{x+1}$.
- (b) Using your sketch, determine a lower bound for a and an upper bound for b .
- (c) Find the values of a and b if $b - a = 1.8$.

(accessible to students on the path to grade 7) [8 marks]