Chapter notes: 13 Vectors

Overview

This first vector chapter focuses on operations with vectors, providing the tools required for the more geometric problems in chapter 14. We recommend approximately eight hours of teaching time.

Introductory problem

The introductory problem highlights some of the difficulties posed by three-dimensional problems. It can be solved using Pythagoras and the cosine rule. But, before applying these rules we need to ask whether the two diagonals cross at all. Vectors will provide a way to answer this question without having to rely on our spatial intuition. The worked solution is given at the end of the chapter, page 409; the idea being that students should be able to answer the question using the methods covered in the chapter.

13A Positions and displacements, p375

This section introduces the concept of vectors as 'arrows', the graphical representation of vector operations, and the representation of vectors using components. The distinction is made between 'vector displacements' and 'position vectors'; this distinction is important for understanding problems involving equations of lines in chapter 14. Worked example 13.4 and question 10 show how vectors can be used to describe geometrical properties; this application of vectors is emphasised in the new syllabus.

13B Vector algebra, p384

In this section we look at vectors in a more abstract way, emphasise that the same algebraic rules apply no matter what the vectors represent. We also look at the connection between algebraic and diagrammatic representations, which is really important for dealing with lines and planes problems in the next chapter.

13C Distances, p389

In this section we return to vectors describing positions and displacements. The diagonal of a cube links back to Key point 11.7 on p323; you can use this to discuss different approaches to solving three-dimensional problems.

The 'From another perspective' box mentions the idea that not all quantities can be ordered. This issue also comes up with complex numbers.

Unit vectors are not used very much in this course. However, the ability to find a vector of a given length in a given direction, as in Worked example 13.7 (b), is useful.

Hints for the grade 7 questions:

11. Write $\overrightarrow{AB}$ in terms of t. You may want to ask students to think about all possible positions of point B (they form a straight line).

12. Write $\overrightarrow{PQ}$ in terms of t. Note that you can minimise the square of the distance rather than the distance itself. If you discussed the straight line interpretation in question 11, then this question can be interpreted as finding the shortest distance from a point to a line. We will meet this again in Worked example 14.8.

13D Angles, p393

We have introduced the scalar product as the quantity $a_1b_1 + a_2b_2 + a_3b_3$, which comes up when we try to calculate an angle between vectors (see Key points 13.6 and 13.7). The other way of defining it is as $|\boldsymbol{a}||\boldsymbol{b}| \cos \theta$ and then showing how to calculate it using components. In the next section we will mention the second definition and look at properties of the scalar product as an algebraic operation.

The derivation of the formula in Key point 13.6 is not too difficult – it may be worth going through. (See Fill-in proof sheet 12, which you could make even simpler by looking at the two-dimensional case. The questions and the 'Theory of knowledge issues' box at the end of the proof sheet explore the idea of definition in mathematics.)

13E Properties of the scalar product, p397

This section looks at the scalar product as an algebraic operation and compares its properties to 'normal' multiplication. The special case of perpendicular vectors will be used widely in the next chapter.

The 'Theory of knowledge issues' box brings up the question of fourth dimension. Some students may have encountered the idea of the fourth dimension being time (which comes from Einstein's Theory of Relativity). But they could also be encouraged to think of dimension simply as the number of coordinates required to define a position of a point.

Hints for grade 7 questions:

12. (b) You need to use the result $\boldsymbol{a} \cdot \boldsymbol{a} = |\boldsymbol{a}|^2$.

13. (a) You don't need to find the equation of the line AB, it is sufficient to show that $\overrightarrow{OB}$ is parallel to $\overrightarrow{OA}$.

(b) Express $\overrightarrow{BC}$ and $\overrightarrow{BA}$ in terms of λ.

13F Areas, p402

As with the scalar product, we have introduced the vector product through its use for finding areas. Notice that we have not yet said anything about the direction of the resulting vector, we are just using its magnitude.

Hints for grade 7 questions:

5. (a) Use the fact that, for example, $\overrightarrow{AD} = \overrightarrow{BC}$.

13G Properties of vector product, p404

This section looks at the vector product as a vector. You may want to show that $\boldsymbol{a} \times \boldsymbol{b}$ is perpendicular to both $\boldsymbol{a}$ and $\boldsymbol{b}$ by direct calculation, either on a specific example or in the general case. The use of the vector product to find a vector perpendicular to two given directions will be essential when working with equations of planes in the next chapter.

The 'Research explorer' on page 405 mentions the right-hand rule, which can be used to remember the direction of the vector product. The right-hand rule says that, if $\boldsymbol{a} \times \boldsymbol{b} = \boldsymbol{c}$ and we hold the thumb, index finger and middle finger of the right hand so that they are perpendicular to each other, then the thumb points in the direction of $\boldsymbol{a}$, the index finger in the direction of $\boldsymbol{b}$ and the middle finger in the direction of $\boldsymbol{c}$. Some applications mentioned are:

- $\boldsymbol{F} = q\boldsymbol{v} \times \boldsymbol{B}$, where $\boldsymbol{F}$ is the magnetic force on charge q moving with velocity $\boldsymbol{v}$ in the magnetic field $\boldsymbol{B}$
- the velocity of an object performing circular motion, given by $\boldsymbol{v} = \boldsymbol{\Omega} \times \boldsymbol{r}$ where $\boldsymbol{\Omega}$ is a vector in the direction of the axis of rotation and $\boldsymbol{r}$ is the position vector from the centre
- the (turning) moment of a force $\boldsymbol{F}$ acting at a point with position vector $\boldsymbol{r}$, given by $\boldsymbol{M} = \boldsymbol{r} \times \boldsymbol{F}$, with applications in statics and fluid dynamics.

The idea in the 'From another perspective' box on page 406, about two different ways of interpreting multiplication, could be discussed before introducing scalar and vector product, for example when talking about operations with vectors. The scalar product could be related to projections, although this is not on the syllabus any more.

Hints for grade 7 questions:

7. Use the definition of scalar and vector products in terms of $|\mathbf{a}|$, $|\boldsymbol{b}|$ and θ.

