

## Chapter notes: 4 Algebraic structures

### Overview

Although several sections of this chapter do not map directly onto the syllabus, it focuses on some core skills which are needed across all other chapters. It is recommended that the chapter is covered after chapters 2 and 3, so that sufficiently complex structures are available. We would recommend six hours of teaching time.

### Introductory problem

This introduces the idea that although equations and identities look very similar, they can be treated in very different ways. The worked solution is given at the end of the chapter, page 115; the idea being that students should be able to answer the question using the methods covered in the chapter.

### 4A Solving equations by factorising, p87

An example for the ‘Research explorer’ comment might be  $xy - x - y + 1 = 37$ .

*Hints for the grade 7 questions:*

4. How can an expression of the form  $a^b$  equal 1?

### 4B Solving equations by substitution, p89

*Hints for the grade 7 questions:*

4. Use the change of base formula to get all logarithms to base 2.

### 4C Features of graphs, p91

It is vital that students are competent at using the graphing functions on their calculator, especially being able to view graphs in appropriate scales and interpret artefacts from the graphs.

*Hints for the grade 7 questions:*

4. This question deliberately makes the choice of scales difficult. Separate scales will be needed in positive and negative domains.

### 4D Using a graphical calculator to solve equations, p94

*Hints for the grade 7 questions:*

3. Sketch the graph of the described situation. Rearrange each new equation into the form  $\ln x = \dots$  and consider the new gradient.

### 4E Solving simultaneous equations by substitution, p97

Many students may have already met this topic, but the link with the quadratic discriminant and applications to logarithms and exponents may be new.

*Hints for the grade 7 questions:*

6. A square number is always positive.

### 4F Systems of linear equations, p101

This section replaces the matrix methods which were used in the previous specification.

### 4G Solving inequalities, p110

Answer the questions using a mixture of calculator and non-calculator methods. Question 2 emphasises the dangers of dividing by a quantity without knowing whether it is positive or negative.

### 4H Working with identities, p113

An example to show that reduction to a true statement is a logically-flawed proof method might be:

$$1 = 3$$

Subtract 2:

$$-1 = 1$$

Square:

$$1 = 1$$

*Hints for the grade 7 questions:*

2. Start with the fact that the coefficient of  $x^2$  is zero.