

Chapter notes: 18 Further differentiation methods

Overview

This chapter extends the number of functions which can be differentiated. In each section there are also questions on optimisation and graphical interpretations of derivatives. We estimate that this will need eight teaching hours.

Introductory problem

By considering the cone when θ is very small and very close to 90° , it should be clear that there is some maximum volume. However, if an expression for the volume is created, it should become apparent that it cannot be differentiated using the methods from chapter 16. The worked solution is given at the end of the chapter, page 619; the idea being that students should be able to answer the question using the methods covered in the chapter.

18A Differentiating composite functions using the chain rule, p599

We have placed this section after trigonometric and exponential differentiation to avoid a common misconception that the chain rule only applies to functions of the form $(f(x))^n$. Often students learn a chain rule method without actually knowing that the chain rule is that stated in Key point 18.1.

The equation of the catenary, referred to in question 9, is normally of the form $y = e^x + e^{-x}$. This can lead to an interesting investigation of hyperbolic functions.

Hints for grade 7 questions:

9. (b) Rewrite $\frac{1}{e^{2x}}$ as e^{-2x} . When the derivative is set to zero, the result is a disguised cubic.

18B Differentiating products using the product rule, p604

Hints for grade 7 questions:

11. You will need to express $\sqrt{1+x}$ as $\frac{1+x}{\sqrt{1+x}}$.

12. (a) Write x in the form $e^{f(x)}$ then use the rules of exponents.

18C Differentiating quotients using the quotient rule, p607

Hints for grade 7 questions:

7. You will need to express $\sqrt{x+1}$ as $\frac{x+1}{\sqrt{x+1}}$.

8. You will need to show that $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} > 0$ at $x = a$, given that $f'(a) = 0$ and $f''(a) < 0$.

Apply the quotient rule twice to find $\frac{d^2y}{dx^2}$.

18D Implicit differentiation, p611

Although the Cartesian equation of the circle is not required for the IB syllabus, it is an useful example of implicit equations.

Hints for grade 7 questions:

10. (d) To solve the cubic equation you can use the fact that $x = 4$ at the point of tangency; i.e. use it as a factor of the cubic equation.

18E Differentiating inverse trigonometric functions, p616

Hints for grade 7 questions:

6. Use the fact that a square number is always positive to show that $y'' > 0$. (Points of inflexion require $y'' = 0$.)