Self-assessment answers: 4 Algebraic structures

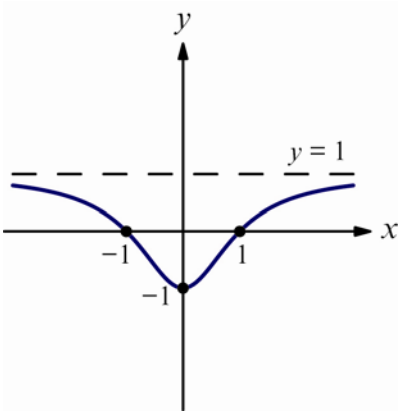
1. $e^{2x} + 2e^{x} - 15 = 0$

$\Rightarrow$ $(e^{x} + 5)(e^{x} - 3) = 0$

$\Rightarrow$ $e^{x} = -5$ or 3. Reject the negative solution.

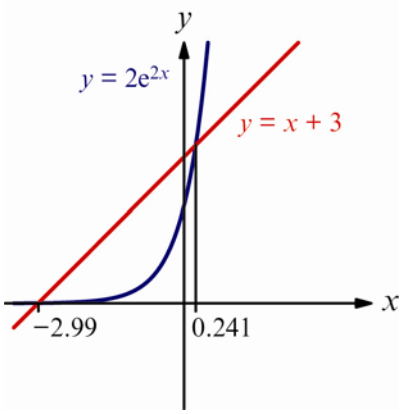
$\Rightarrow$ $x = \ln 3$ *[5 marks]*

2.



[4 marks]

3. Using a GDC:



So the interval for which $2e^{2x} \leq x + 3$ is $[-2.99, 0.241]$ *[4 marks]*

4. $x^2 + 1 = 5 - x$

$\Rightarrow \quad x^2 + x - 4 = 0$

$\Rightarrow \quad x = \dfrac{-1 \pm \sqrt{17}}{2}$

$\Rightarrow \quad \left(\dfrac{-1+\sqrt{17}}{2}, \dfrac{11-\sqrt{17}}{2}\right), \left(\dfrac{-1-\sqrt{17}}{2}, \dfrac{11+\sqrt{17}}{2}\right)$ are the points of intersection. *[6 marks]*

5. (a) $\begin{cases} x - 2y + z = 5 & (1) \\ 2x + y - z = -1 & (2) \\ x + 8y - 5z = k & (3) \end{cases}$

(1) + (2) $\Rightarrow \quad 3x - y = 4 \qquad (4)$

(3) + 5(1) $\Rightarrow \quad 6x - 2y = 25 + k \qquad (5)$

(5) – 2(4) $\Rightarrow \quad 17 + k = 0$

$\Rightarrow \quad k = -17$

(b) From the above, we see that (3) = 2(2) – 3(1); the three planes will have a linear intersection.

(4) gives that $y = 3x - 4$

Let $x = t$. Then $y = 3t - 4$.

(1) then gives $z = 5 + 2y - x = 5t - 3$

The general solution is:

$$\left.\begin{array}{l} x = t \\ y = 3t - 4 \\ z = 5t - 3 \end{array}\right\} \text{for } t \in \mathbb{R}$$

[11 marks]