Chapter notes: 1 Counting principles

Overview

This is a relatively small section in the algebra topic (30 hours), and normally it is tested with one short question. However, it is often found amongst the harder topics in the Core syllabus. We would recommend approximately three hours spent on this topic.

Introductory problem

The purpose of this problem is to convey how big some of the numbers in counting can get. Initially it is intended to be estimated rather than calculated. The worked solution is given at the end of the chapter, page 24; the idea being that students should be able to answer the question using the methods covered in the chapter.

1A The product principle and the addition principle, p1

The terms 'product principle' and 'addition principle' are not universal or important to remember. The emphasis in this section is on breaking up problems into smaller parts. It is often useful to do this orally, although it may be difficult at first.

The 'Research explorer' in this section refers to infinite sets; the results of the investigation are not what the student might expect. For example, it is a fact that there are the same number of positive even numbers as there are positive whole numbers. This can be illustrated using a one-to-one correspondence between them:

$$1 \longrightarrow 2$$

$$2 \longrightarrow 4$$

$$3 \longrightarrow 6$$

$$\vdots$$

$$n \longrightarrow 2n$$

Cantor showed that there is no one-to-one correspondence between the positive integers and the decimals, so concluded that there was a 'bigger' infinity of irrational numbers.

1B Counting arrangements, p6

This section is all about permutations and the factorial function. The 'From another perspective' box refers to the fact that some textbooks use 'permutation' to mean arrangements of the whole set whilst others use 'permutation' or 'n-permutation' to mean arrangements of a subset.

The exam-style questions in this section involve counting with relatively simple constraints. They are meant to be seen as an application of section 1A as well as 1B. Some people may find the last question, question 10, very easy if they look at it in the right way. This highlights the importance of looking at problems flexibly. You may like to return to the question after working on combinations to see how approaching it by thinking about it explicitly as three rows of ten students gives the same answer.

1C Algebra of factorials, p9

This section is not formally mentioned on the syllabus, so you may wish to omit it if time is limited. However, it has appeared in past examination papers, particularly in the statistics option.

The 'Research explorer' on page 10 encourages thoughts about whether mathematics is driven by definitions, or usefulness drives the mathematics. 0! may be shown to equal one by considering the pattern formed by decreasing factorials, or by considering how many ways there are to arrange no objects. The Gamma function is possible to work with once integration by parts has been studied, although some intuition about improper integrals will be required.

Take note that question 6 in this section results in a cubic equation which needs to be solved using calculator methods.

1D Counting selections, p11

From an exam perspective, this is probably the most important section. Throughout the chapter, we have tried to be consistent in using 'selections' to mean unordered subsets and 'arrangements' to mean ordered subsets (see section 1B), but this is not official IB terminology.

The 'Theory of knowledge issues' box in this section (page 12) highlights the idea that labels can sometimes appear to obstruct knowledge. But, hopefully they allow more complex ideas to be developed.

If section 1C has been covered, then question 3 should be answered by turning the equations into a quadratic or cubic equation. Otherwise it must be approached using trial-and-improvement or using the calculator to form a table.

1E Exclusion principle, p15

Students may find the application of this principle fairly obvious. What needs to be emphasised is how to recognise questions where it is a useful method.

1F Counting ordered selections, p18

The formula for permutations is new to the course. We have derived it from the process of selecting a subset and then arranging it. However, you might like to show how the formula can be derived directly from the product rule. The main emphasis in this topic should be distinguishing between situations using combinations and situations using permutations.

1G Keeping objects together or separated, p21

This section is aimed at students aiming to get a high grade in the examination. It may be better to avoid it with weaker students.

Mixed examination practice 1, p25

Short question 7 requires ideas from section 1C.

Long question 4 encourages a counting way of looking at a question which is not obviously about combinations. The R's and D's correspond to moves right and down.

Long question 5 tests the idea of double counting. $\binom{12}{6}$ would count the same six people on a team twice: once when all six were selected, and once when all six were not selected.

