

## Self-assessment answers: 5 The theory of functions

1. (a) Domain  $\mathbb{R}$ ; Range  $[2, \infty[$

(b) Domain  $x \in \mathbb{R}, x \neq -3$ ; Range  $y \in \mathbb{R}, y \neq 1$

(c) Domain  $x \in \mathbb{R}, x \neq -2, 1$ ; Range  $y \in \mathbb{R}, y \notin ]-0.5, 3[$

[7 marks]

2.  $f$  has domain  $x > \frac{2}{3}$  and range  $\mathbb{R}$ , so  $f^{-1}$  has domain  $\mathbb{R}$  and range  $]2/3, \infty[$

$$f(x) = \ln(3x - 2), \Rightarrow e^{f(x)} + 2 = 3x$$

$$\Rightarrow x = \frac{e^{f(x)} + 2}{3}$$

$$\Rightarrow f^{-1}(x) = \frac{e^x + 2}{3}, x \in \mathbb{R}$$

[3 marks]

3.  $f g(x) = f(x + 2) = 2(x + 2)^2 - 11$

$$2(x + 2)^2 - 11 = 2x \Rightarrow 2x^2 + 6x - 3 = 0$$

$$\Rightarrow x = \frac{-6 \pm \sqrt{60}}{4} = \frac{-3 \pm \sqrt{15}}{2}$$

[5 marks]

4. (a) (i)  $\left(\frac{a}{3}, 0\right), \left(0, \frac{a}{b}\right)$

(ii)  $y = 3, x = b$

(iii)  $y \in \mathbb{R}, y \neq 3$

(b)  $\frac{3x-a}{x-b} = 2 \Rightarrow 3x-a = 2x-2b$

$\Rightarrow x = a - 2b$

(c)  $f(x) = \frac{3x-a}{x-b} \Rightarrow (x-b)f(x) = 3x-a$

$\Rightarrow x(f(x)-3) = bf(x)-a$

$\Rightarrow x = \frac{bf(x)-a}{f(x)-3}$

$\Rightarrow f^{-1}(x) = \frac{bx-a}{x-3}$ . Domain of  $f^{-1}$  is range of  $f$ .  $x \in \mathbb{R}, x \neq 3$

(d) For  $f(x)$  to be the same as  $f^{-1}(x)$ ,  $b = 3$ .

[15 marks]