

Chapter notes: 7 Sequences and series

Overview

In sections 7E and 7F, logarithms will be used. If this has not been taught, these questions can still be done using the calculator to solve equations. It needs approximately five hours of teaching time.

Introductory problem

This problem illustrates applications to the real world. It also gives some insight into how fast geometric growth can be. Students might like to estimate the answer at this stage, then come back to the problem after looking at geometric series. The worked solution is given at the end of the chapter, page 214; the idea being that students should be able to answer the question using the methods covered in the chapter.

7A General sequences, p190

There is often confusion between a term's position, n , and its value, u_n .

In the 'Research explorer' box on page 191, the link between the Fibonacci sequence and the golden ratio comes from the formula for the sequence:

$$F^n = \frac{\phi^n - \phi'^n}{\phi - \phi'}$$

where, $\phi = \frac{1+\sqrt{5}}{2}$ and $\phi' = \frac{1-\sqrt{5}}{2}$.

7B General series and sigma notation, p193

Students are often intimidated by sigma notation. In the IB, questions requiring manipulation of expressions with sigma notation are usually not required – students need only be able to understand what it means. One major teaching point is the difference between dummy variables and real variables.

7C Arithmetic sequences, p195

Hints for the grade 7 questions:

8. The number of digits goes up in an arithmetic sequence between pages 1 and 9 then a different arithmetic sequence between pages 10 and 99, and another between pages 100 and 999.

7D Arithmetic series, p199

This work often links to work on inequalities (section 4G). The hardest questions do not make it obvious that an arithmetic sequence is involved at all (e.g. question 11).

Hints for the grade 7 questions:

10. Set up two simultaneous equations to find u_1 and d .
11. Numbers which are multiples of both 21 and 14 are multiples of 42. Find the sum of all relevant multiples of 42.

7E Geometric sequences, p201

We recommend that you cover logarithms before looking at this section. Without logarithms, numerical methods or a GDC will be required. Worked example 7.14 deals with a situation which can cause confusion. It does not often appear in examinations, so you might wish to look at it only briefly.

In the ‘Theory of knowledge issues’ box on page 204, it might be interesting to show students familiar with complex numbers that $\log(-x) = \log x \pm i\pi$ is a possible interpretation.

Hints for the grade 7 questions:

10. Equations can be formed by finding expressions for the common ratios and differences.

11. Consider S_1 and S_2 to find the first two terms of the arithmetic sequence.

7F Geometric series, p205

In Worked example 7.16, the common ratio is greater than one so dividing both sides of the inequality by $\log r$ does not cause an issue. You might like to remind students that if r was less than one, we would have to be more careful.

7G Infinite geometric series, p207

There are many philosophical problems dealing with convergence, of which the most famous are Zeno’s paradoxes.

In the ‘Theory of knowledge issues’ box on page 208, there is reference to different groupings of the terms of a geometric sequence:

$$S_{\infty} = a - a + a - a + a - a \dots$$

One grouping suggests:

$$\begin{aligned} S_{\infty} &= (a - a) + (a - a) + (a - a) \dots \\ &= 0 + 0 + 0 \dots \\ &= 0 \end{aligned}$$

Another grouping is:

$$\begin{aligned} S_{\infty} &= a + (-a + a) + (-a + a) + (-a \dots \\ &= a + 0 + 0 + 0 \dots \\ &= a \end{aligned}$$

The grouping which students often find most confusing is when the sequence is rewritten twice, slightly offset:

$$S_{\infty} = a - a + a - a + a - a \dots$$

$$S_{\infty} = \quad a - a + a - a + a - a \dots$$

Adding these together we get:

$$2S_{\infty} = a + 0 + 0 + 0 \dots$$

So,
$$S_{\infty} = \frac{a}{2}$$

This highlights the danger of doing something as simple as grouping with infinite series.

Hints for the grade 7 questions:

10. ERRATA: This question should read 2^x instead of $2x$.

You may like to use a graph to determine when 2^x is less than 1.

11. The trap to avoid is to use the formula inappropriately. The function is only convergent in one part.

7H Mixed questions, p211

The new syllabus has a greater emphasis on applications, so you might expect to see more questions of this style.

Hints for the grade 7 questions:

7. There are two geometric series here – the heights on the way up and on the way down. You may want to treat them separately.

8. Students should recognise the result in (a) as a geometric series.