Self-Discovery Worksheet 1 An introduction to logarithms

A

Here are some examples of a new function called a logarithm. Describe the function.

(a) $\log_{10}(10) = 1$ (b) $\log_{10}(100) = 2$ (c) $\log_{10}(1000) = 3$

(d) $\log_{10}(1\,000\,000) = 6$ (e) $\log_{10}\left(\frac{1}{10}\right) = -1$ (f) $\log_{10}(0.01) = -2$

(g) $\log_2(4) = 2$ (h) $\log_2(16) = 4$ (i) $\log_3(27) = 3$

(j) $\log_5(125) = 3$ (k) $\log_8 64 = 2$ (l) $\log_7\left(\frac{1}{49}\right) = -2$

(m) $\log_{15}(1) = 0$ (n) $\log_5\left(\sqrt{5}\right) = \frac{1}{2}$ (o) $\log_8(2) = \frac{1}{3}$

B

Where possible find the unknown in each of the following.

(a) $\log_{10}(10\,000) = x$ (b) $\log_4(16) = x$ (c) $\log_3(9) = x$

(d) $\log_5 x = 3$ (e) $\log_3 x = 4$ (f) $\log_2 x = 8$

(g) $\log_4 x = 3$ (h) $\log_5 x = -2$ (i) $\log_x(49) = 2$

(j) $\log_x(121) = 2$ (k) $\log_x\left(\frac{1}{27}\right) = -3$ (l) $\log_x(32) = 5$

(m) $\log_4 x = 0$ (n) $\log_x(1) = 0$ (o) $\log_x(125) = -3$

(p) $\log_{32}(64) = x$ (q) $\log_4(-2) = x$

C

Where necessary use your calculator to evaluate to 3 decimal places.

(a) $\log_{10} 2.34$, $\log_{10} 100, \log_{10} 234$

(b) $\log_{10} 18.34, \log_{10} 1000, \log_{10} 18340$

(c) $\log_{10} 3, \log_{10} 12, \log_{10} 36$

(d) $\log_{10} 8, \log_{10} 2, \log_{10} 16$

(e) $\log_2 8, \log_2 4, \log_2 32$

(f) $\log_3 27, \log_3 3, \log_3 81$

D

On the basis of C make a hypothesis for the relationship between $\log_n x$, $\log_n y$ and $\log_n(xy)$.

E

Make and test hypotheses for $\log\left(\frac{x}{y}\right)$, $\log(x+y)$, $\log\frac{1}{y}$ and $\log x^p$.

Self-Discovery Worksheet 2 Changing functions and their graphs – an investigation

This investigation is designed to discover more about how the graph you plot is affected by changing the function. You may wish to use a computer graphing package or your graphical calculator.

For this investigation we shall start by using the function $f(x) = x^2 - x - 2$ plotted in the region $-5 \le x \le 5$ and $-10 \le y \le 10$.

1. First plot the graph $y = f(x)$ and highlight it in some way (make it bold, or dashed, or change its colour). Make a note of where it crosses the axes.

2. (a) Investigate what happens when you change $f(x)$ by adding or subtracting a constant from the whole function. You might want to plot the following functions.

Make sure you only have two graphs (the original and one of the ones below) on display at any one time.

(i) $y = x^2 - x - 2 + 4$

(ii) $y = x^2 - x - 2 - 2$

(iii) $y = f(x) + 4$

(iv) $y = f(x) - 3$

(v) $y = x^2 - x + 5$

(vi) $y = x^2 - x - 1$

> Take care when examining these graphs. You must use both perception (what they look like) and logic (an examination of, for example, how the axis intercepts change). They may not necessarily match up; so which one do you trust more?

(b) Suggest a rule for what happens when you change the graph $y = f(x)$ to $y = f(x) + c$.

Does your rule work for other functions?

3. (a) Investigate what happens when you change $f(x)$ by adding or subtracting a constant from the argument of the function. You might want to plot the following functions.

Make sure you only have two graphs (the original and one of the ones below) on display at any one time.

(i) $y = (x+1)^2 - (x+1) - 2$

(ii) $y = (x-3)^2 - (x-3) - 2$

(iii) $y = f(x+2)$

(iv) $y = f(x-2)$

(v) $y = x^2 + 5x + 4$

> **EXAM HINT**
>
> It is not obvious how part (v) is algebraically related to the original function. Once you have sketched the graph you might want to rewrite it in a form which makes the transformation more obvious. These are the hardest type of transformation questions.

(b) Suggest a rule for what happens when you change the graph $y=f(x)$ to $y=f(x+d)$.

Does your rule work for other functions?

4. (a) Investigate what happens when you change $f(x)$ by multiplying or dividing the whole function by a constant. You might want to plot the following functions.

Make sure you only have two graphs (the original and one of the ones below) on display at any one time.

(i) $y=3(x^2-x-2)$

(ii) $y=\frac{1}{4}(x^2-x-2)$

(iii) $y=4f(x)$

(iv) $y=\frac{x^2}{2}-\frac{x}{2}-1$

(v) $y=-(x^2-x-2)$

(b) Suggest a rule for what happens when you change the graph $y=f(x)$ to $y=pf(x)$.

Does your rule work for other functions?

5. (a) Investigate what happens when you change $f(x)$ by multiplying or dividing the argument of the function by a constant. You might want to plot the following functions.

Make sure you only have two graphs (the original and one of the ones below) on display at any one time.

(i) $y=(2x)^2-(2x)-2$

(ii) $y=\left(\frac{1}{2}x\right)^2-\left(\frac{1}{2}x\right)-2$

(iii) $y=f(3x)$

(iv) $y=f\left(\frac{1}{3}x\right)$

(v) $y=16x^2-4x-2$

(vi) $y=\frac{x^2}{25}-\frac{x}{5}-2$

(vii) $y=f(-x)$

(b) Suggest a rule for what happens when you change the graph $y=f(x)$ to $y=f(qx)$.

Does your rule work for other functions?

5. Summarise your observations using the following table:

Change to the function	Change to the graph
$f(x)+c$	
$f(x+d)$	
$pf(x)$	
$f(qx)$	
$-f(x)$	
$f(-x)$	

Self-Discovery Worksheet 3 Investigating derivatives of polynomials

You may want to use a spreadsheet and a graphing programme or a graphical calculator for this investigation.

1. Draw the graph of $y = x^2$ for $0 \le x \le 4$ and $0 \le y \le 10$ and mark on the point $A(1,1)$.

(a) Plot the point $B_1(3,9)$;

draw on the line AB_1 and calculate its gradient.

(b) Plot the point $B_2(2,4)$;

draw on the line AB_2 and calculate its gradient.

(c) Plot the point $B_3(1.5,2.25)$;

draw on the line AB_3 and calculate its gradient.

(d) Sketch in a line with B very, very close to A.

Does it look like a special type of line you have met before?

(e) Investigate numerically what happens to the gradient of AB as B gets closer to A?

(f) Write an expression for the gradient of AB when B has got an x-coordinate of $1+h$.

Can you use this expression to justify the result you found in part (e)?

2. Repeat the investigation in question 1 for different positions of A.

Can you come up with a general rule?

3. Try repeating the procedure of questions 1 and 2 with functions other than $y = x^2$.