Chapter notes: 16 Basic differentiation and its applications

Overview

This chapter is fundamental to all of the other calculus chapters. It begins by developing a graphical interpretation of derivatives, then it builds up a reasonable range of functions which can be differentiated. Finally, these are applied to the common problems of optimisation and finding tangents. We think approximately 12 hours of teaching time is required.

Introductory problem

This problem is designed to result in the need for optimisation. You might like to use it as an opportunity to get students to think about other situations where optimisation is desired. The worked solution is given at the end of the chapter, page 566; the idea being that students should be able to answer the question using the methods covered in the chapter.

16A Sketching derivatives, p527

This section introduces derivatives graphically. As well as developing an understanding of the topic, several examination questions have tested this skill. Some interesting examples to use in question 3 might be:

	Situation when true	**Situation when false**
(a)	$y = x^2, x > 0$	$y = x^3, x < 0$
(b)	$y = \frac{1}{x}, x < 0$	$y = x^2, x < 0$
(d)	$y = x^2, x = 0$	$y = x^3, x = 0$
(e)	$y = e^x$	$y = -e^{-x}$
(f)	$y = x^2$	$y = \|x\|$

16B Differentiation from first principles, p535

Hints for grade 7 questions:

6. Use the same difference of two squares idea as seen in Worked example 16.4.

16C Rules of differentiation, p538

There are no specific teacher notes for this section.

16D Interpreting derivatives and second derivatives, p541

Hints for grade 7 questions:

10. If the *gradient* is increasing then $\frac{d^2y}{dx^2} > 0$.

16E Trigonometric functions, p547

If x is measured in degrees then, $\frac{\mathrm{d}}{\mathrm{d}x}(\sin x) = \frac{\pi}{180}\cos x$. It might be useful to get the stronger students to prove this.

16F The exponential and natural logarithm functions, p549

One definition of the value e is given by Key point 16.7. Another is given in section 2C. A third comes from the series definition that $\mathrm{e} = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!}$ ... . You might like to use this as an opportunity to discuss how mathematicians have some freedom to choose what their fundamental definitions are.

Question 5 provides good revision of the rules of logarithms and indices.

16G Tangents and normals, p550

The $y - y_1 = m(x - x_1)$ version of the equation for a straight line (covered in Prior learning section R) will be extremely useful in this section.

Hints for grade 7 questions:

7. Use the fact that the gradient is $\tan\theta$ where θ is the angle with the positive x-axis.

8. Find the equation of the tangent at $x = a$, then find the coordinates of P and Q.

9. Find the equation of the tangent and solve simultaneously with the original curve. Remember that you do not need to find all solutions to the resulting cubic.

16H Stationary points, p554

When asked to justify the nature of stationary points, it is fine to use the second derivative or to find the sign of the derivative at points close to the stationary point.

Hints for grade 7 questions:

9. Use the $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ method for determining the stationary point.

16I General Points of inflexion, p560

Hints for grade 7 questions:

5. First solve $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 0$.

6. $f''(x) = 0$ corresponds to a stationary point on the graph of $f'(x)$.

16J Optimisation, p562

This section will be covered again with more challenging applications in section 20D. However, we thought that it was useful to show a major application of calculus at this early stage. Students frequently forget to check end points or poles for the global maximum and minimum.

Hints for grade 7 questions:

9. (b) This is asking you to maximise $\frac{\mathrm{d}V}{\mathrm{d}t}$, so solve $\frac{\mathrm{d}^2V}{\mathrm{d}t^2} = 0$.

12. (c) Solve a cubic inequality.

(d) Form a new function as the difference between energy production and energy usage.

