Revision answers: Statistics and probability (Topic 5)

Coursebook chapters: 21–24

1. Use the midpoint to create a frequency table:

Midpoint	Frequency
9	26
14.5	18
19.5	45

From GDC: mean=15.4, S.D. = 4.54 *[4 marks]*

2. $X \sim \text{Po}(18)$, $\text{P}(X > 20) = 1 - \text{P}(X \leq 20) = 0.269$ *[4 marks]*

3. (a) $\int_k^{k+2} x\left(\frac{x-k}{2}\right)\text{d}x = \left[\frac{x^3}{6} - \frac{kx^2}{4}\right]_k^{k+2} = 3$, solving the equation gives $k = \frac{5}{3}$.

(b) $\text{Var}(X) \int_{5/3}^{11/3} \frac{x^2\left(x - \frac{5}{3}\right)}{2}\text{d}x - 3^2 = \frac{2}{9}$ *[6 marks]*

4. (a) P(walk) = 1 − P(rain or late) = 1 − (P(rain) + P(late) − P(rain and late))

As they are independent, P(rain and late) = 0.3 × 0.6 = 0.18, so P(walk) = 0.28

(b) $\text{P}(\text{rain}|\text{bus}) = \frac{\text{P}(\text{rain and bus})}{\text{P}(\text{bus})} = \frac{\text{P}(\text{rain})}{\text{P}(\text{bus})} = \frac{0.3}{0.72} = 0.417$ *[6 marks]*

5. $X \sim \text{N}(0.3, 0.04^2)$

(a) $\text{P}(X > 0.36) \times \text{P}(X > 0.36) = 0.00446$

(b) $2 \times \text{P}(X < 0.4) \times \text{P}(X > 0.4) = 0.0123$ *[6marks]*

6. $\int_a^{2a} 5 - 2x \, dx = 1 \Rightarrow 3a^2 - 5a + 1 = 0 \Rightarrow a = 0.232$ or 1.43

But $5 - 2x \geq 0$ for $a \leq x \leq 2a$, so $a = 0.232$. *[5 marks]*

7. (a) $X \sim B(4, p)$, $P(X = 3 \text{ or } 4) = 4p^3(1-p) + p^4 = 4p^3 - 3p^4$

(b) $4p^3 - 3p^4 = 0.05$ when $p = 0.248$ (using GDC) *[5 marks]*

8. (a) $E(X) = 0 \times \frac{1}{2} + 1 \times \frac{1}{3} + 2 \times \frac{1}{6} = \frac{2}{3}$

(b) The possible ways of scoring three points out of four hands are (CHECK THE TABLE!):

Distribution of points	**Combinations**	**Probability**
2, 1, 0, 0	$4 \times 3 = 12$	1/72
1, 1, 1, 0	4	1/54

Out of those, only the ones in the first row contain a two-point hand, so:

$$p = \frac{12 \times \frac{1}{72}}{12 \times \frac{1}{72} + 4 \times \frac{1}{54}} = \frac{9}{13} \text{ or } 0.69$$ *[7 marks]*

9. $X \sim N(252, \sigma^2)$, $P(X \geq 250) = 0.99$ so $P(X < 250) = 0.01$

Standardising: $P\left(Z < \frac{250 - 252}{\sigma}\right) = 0.01 \Rightarrow -\frac{2}{\sigma} = -2.32 \Rightarrow \sigma = 0.860$ g *[5 marks]*

10. $V \sim Po(3)$, $P(V \geq 5) = 1 - P(V \leq 4) = 0.185$

$S \sim B(10, 0.7)$, $P(S \geq 8) = 1 - P(S \leq 7) = 0.383$

P(exactly one won) $= 0.185 \times (1 - 0.383) + 0.383 \times (1 - 0.185) = 0.426$

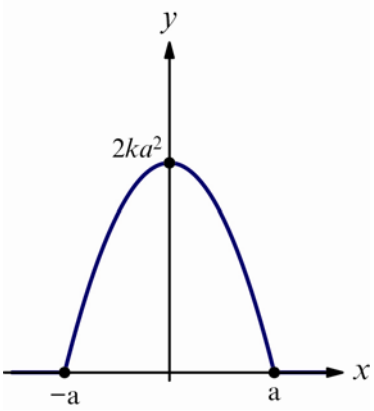
$$P(\text{Vesna won}) = \frac{0.185 \times (1 - 0.383)}{0.426} = 0.268$$ *[7 marks]*

11. $X \sim \text{N}(50, 4)$, P(double-yolked) = $0.1 \times \text{P}(X > 60) = 0.000210$

$Y \sim \text{B}(12, 0.000210)$, $\text{P}(Y = 1) = 0.00740$ *[5marks]*

12. (a) There are two possibilities: $|x^2 - a^2| = x^2 - a^2$ (when $x^2 \geq a^2$) or $|x^2 - a^2| = a^2 - x^2$ (when $x^2 < a^2$). So:

$$f(x) = \begin{cases} 0 & \text{when } x^2 \geq a^2 \\ 2k\left(a^2 - x^2\right) & \text{when } x^2 < a^2 \end{cases}$$



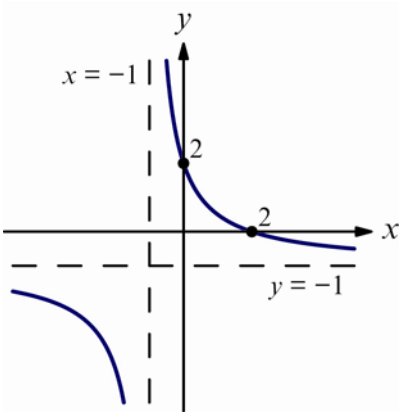
Note that you could sketch the graph on the GDC by using a particular value of a.

(b) $[-a, a]$ (accept] $-a, a[$ as equally valid)

(c) $k\int_{-a}^{a} 2(a^2 - x^2)\text{d}x = 1 \Rightarrow k = \dfrac{3}{8a^3}$ *[8 marks]*

13. $\text{E}(a^x) = \sum_{x=0}^{n} \binom{n}{x} p^x (1-p)^{n-x} a^x = \sum_{x=0}^{n} \binom{n}{x} (ap)^x (1-p)^{n-x} = (ap + 1 - p)^n$ *[5 marks]*

14. (a)



(b) $-1 < a < b \leq 2$, as pdf must be positive.

(c) $\int_a^{a+1.8}\left(\frac{2-x}{x+1}\right)\mathrm{d}x = 1 \Rightarrow 3\ln\left(\frac{a+2.8}{a+1}\right) - 1.8 = 1$ (or use GDC to sketch the integral with variable limits).

Solving gives $a = 0.167$, $b = 1.967$ *[8 marks]*

