

Chapter notes: 24 Continuous distributions

Overview

This chapter extends probability distributions and the methods of chapter 23 into continuous situations through the use of calculus. It also introduces the very important idea of the normal distribution. It requires approximately eight hours of teaching time.

Introductory problem

The introductory problem could generate discussion in different directions:

- Is the information about mean and standard deviation sufficient to determine the distribution of heights?
- How can the distribution of the measured heights be recorded?
- What do we mean by ‘above 10 m’ – has this number been rounded?

Students may already be aware of some of the issues with continuous data, and they may have met histograms as a way of representing continuous distributions. Some students may have also come across the normal distribution in Biology or Geography. The worked solution is given at the end of the chapter, page 788; the idea being that students should be able to answer the question using the methods covered in the chapter.

24A Continuous random variables, p769

The idea of representing the probability by area may be reasonably natural, especially for those who have met histograms before. However, students often struggle with the idea that the probability density function does not have any physical representation. Ask them to think about alternative ways to represent continuous probabilities graphically – they may come up with a cumulative distribution curve (which is not on the syllabus).

The use of $-\infty$ and ∞ as the limits in integrals can be confusing. You should explain that they must be replaced by the actual lowest and highest possible values of the variable, but that some continuous variables can (theoretically) take any real value (you could compare this to the Poisson distribution).

Hints for grade 7 questions:

8. First find the value of k so that the total probability is 1.

24B Expectation and variance of continuous random variables, p773

The ‘From another perspective’ box mentions the link between sums and integrals – a definite integral can be defined as a limit of sums. This is not on the syllabus, but if you have discussed it when introducing definite integration (chapter 17), then you can make the link here. (See the ‘From another perspective’ box on page 583.)

Hints for grade 7 questions:

7. Use integration by parts for definite integrals. Note that $e^{-\frac{x^2}{2}}$ cannot be integrated, so you need to use the result that $\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx = 1$. This is of course the p.d.f. for the normal distribution – it is not required, but can be covered with the strongest students.

24C The normal distribution, p777

The normal distribution tables have been removed from the formula book, so all calculations need to be done on the calculator. Hence standardising is only required when μ or σ are unknown. The worked examples in this section illustrate the amount of working that needs to be shown.

The Z-score still needs to be understood as a measure of how ‘unusual’ a value is. Encourage students to discover the ‘68–95–99.7% rule’ for the amount of data within 1, 2 and 3 standard deviations of the mean; some may already be familiar with it from Biology or Geography.

The ‘From another perspective’ boxes on pages 777 and 780 could generate discussion about how technology has changed the way we do and learn mathematics.

Hints for grade 7 questions:

- 11. (b) This involves conditional probability.
- 12. This combines normal and binomial distributions.
- 13. Use the standardised value (Z-score).
- 14. You will find it helpful to sketch a diagram.

24D Inverse normal distribution, p783

This section is the only place in the Core course where we use data to make inferences about the population. You may want to discuss how reliable these inferences can be; this is the central question in the Statistics option.

The $\Phi(z)$ and $\Phi^{-1}(p)$ notation is not required, but stronger students may find it convenient.

Hints for grade 7 questions:

- 11. It is not necessary to find the mean. Only one equation is required.
- 12. (b) The lower quartile is defined by $P(X \leq Q_1) = 0.25$.
- 13. You will find it useful to sketch a diagram.
- 14. (a) This uses the binomial distribution.

Hint for (Gold) question 15

- 15. Try to make cumulative probabilities correct.
(ERRATA: in the Coursebook the question should say ‘distributed uniformly’, rather than just ‘distributed’.)