Revision answers: Geometry (Topics 3 & 4)

Coursebook chapters: 9–14

1. $\frac{1}{2}r^2\theta - \frac{1}{2}r^2 \sin\theta = \frac{1}{2}(5^2)(1.2 - \sin 1.2) = 3.35 \text{ cm}^2$ *[4 marks]*

2. $\tan(2\theta) = \sqrt{3}$, $0 \le 2\theta \le 360°$

$\arctan\sqrt{3} = 60°$

$\Rightarrow\ 2\theta = 60°$ or $60 + 180 = 240°$

$\Rightarrow\ \theta = 30°, 120°$ *[5 marks]*

3. $\frac{\sin 29}{6.5} = \frac{\sin \hat{A}}{12}$

$\sin \hat{A} = 0.895$

$\therefore\ \hat{A} = 63.5°$ or $180 - 63.5 = 116.5°$ *[5 marks]*

4. $\overrightarrow{AB} = \begin{pmatrix} -3 \\ -5 \\ 0 \end{pmatrix}, \overrightarrow{AC} = \begin{pmatrix} -4 \\ -7 \\ 1 \end{pmatrix}$

$$\cos \hat{A} = \frac{\begin{pmatrix} -3 \\ -5 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} -4 \\ -7 \\ 1 \end{pmatrix}}{\sqrt{3^2 + 5^2 + 0^2}\sqrt{4^2 + 7^2 + 1^2}} = 0.992$$

$$\therefore \ \hat{A} = 7°$$

$\overrightarrow{BA} = \begin{pmatrix} 3 \\ 5 \\ 0 \end{pmatrix}, \overrightarrow{BC} = \begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix}$

$$\cos \hat{B} = \frac{\begin{pmatrix} 3 \\ 5 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix}}{\sqrt{3^2 + 5^2 + 0^2}\sqrt{1^2 + 2^2 + 1^2}} = -0.910$$

$$\therefore \ \hat{B} = 156°$$

$$\hat{C} = 180 - 7 - 156 = 17°$$

[9 marks]

5. amplitude $= \dfrac{5-(-2)}{2} = 3.5 \quad \therefore \ A = 3.5$

$B = \dfrac{5+(-2)}{2} = 1.5$

half-period $= \dfrac{\pi}{2} - \dfrac{\pi}{6} = \dfrac{\pi}{3} \quad \therefore k = \dfrac{\pi}{\frac{\pi}{3}} = 3$

[5 marks]

6. $2x + 30 \in [30, 750]$; $2x + 30 = 60, 300, 420, 660$ $\therefore$ $x = 15, 135, 195, 315$ *[5 marks]*

7. Substitute $x = 5t + 3$, $y = 3t - 1$, $z = t + 4$ into $4x - y + 2z = 7$ to get $t = -\dfrac{14}{19}$

Hence the coordinates are $\left(-\dfrac{13}{19}, -\dfrac{61}{19}, \dfrac{62}{19}\right)$. *[5 marks]*

8. Using $\tan^2 x = \sec^2 x - 1$:

$\sec^2 x - \sec x - 2 = 0 \quad \Rightarrow \quad \sec x = 2 \text{ or } -1$

$\cos x = \dfrac{1}{2}$ or -1, so $x = \dfrac{\pi}{3}, \pi, \dfrac{5\pi}{3}$ *[7 marks]*

9. (a) Write BC = $5x$, AC = $4x$, then:

$$\frac{\sin 2\theta}{5x} = \frac{\sin\theta}{4x} \quad \Rightarrow \quad \frac{2\sin\theta\cos\theta}{5} = \frac{\sin\theta}{4} \quad \Rightarrow \quad 8\cos\theta = 5 \text{ (as } \sin\theta \neq 0\text{)}$$

$\therefore \quad \theta = 51.3°$

(b) $51.3°, 103°, 26.0°$ *[6 marks]*

10. (a) $R\cos(x + \theta) = R\cos\theta\cos x - R\sin\theta\sin x$

$$\Rightarrow \quad R^2 = \left(\sqrt{3}\right)^2 + 1^2 = 4, \ \tan\theta = \frac{1}{\sqrt{3}}$$

$\therefore \ R = 2, \theta = \dfrac{\pi}{6}$

(b) $\dfrac{3}{5 + (-2)} = 1$ *[6 marks]*

11. (a) Gaussian elimination gives:

$$\begin{cases} 4x - y + z = 8 \\ 5x - z = 11 \\ 0 = a - 14 \end{cases}$$

Hence $a = 14$.

(b) Setting $z = t$: $\boldsymbol{r} = \begin{pmatrix} 11/5 \\ 4/5 \\ 0 \end{pmatrix} + t\begin{pmatrix} 1/5 \\ 9/5 \\ 1 \end{pmatrix}$ *[8 marks]*

12. (a) $\boldsymbol{r} = t\begin{pmatrix} 4 \\ -1 \\ -2 \end{pmatrix}$

(b) $(8, -2, -4)$

(c) $\cos\theta = \dfrac{8-5-6}{\sqrt{4+25+9}\sqrt{16+1+4}} = -0.106 \quad \therefore \theta = 96.1°$

(d) $\text{AB} = \sqrt{4^2+1^2+2^2} = \sqrt{21}$, distance $= \sqrt{21}\sin(180 - 96.1) = 4.56$ *[12 marks]*

13. $\sin A = \dfrac{1}{3}$, so $\cos A = \sqrt{1-\left(\dfrac{1}{3}\right)^2} = \dfrac{\sqrt{8}}{3}$ *[3 marks]*

14. (a) $\begin{pmatrix} \cos\theta \\ \sin\theta \\ -\sin\theta \end{pmatrix} \cdot \begin{pmatrix} \cos\theta \\ -\sin\theta \\ \cos\theta \end{pmatrix} = 0$

$\Leftrightarrow \cos^2\theta - \sin^2\theta - \sin\theta\cos\theta = 0$

$\Leftrightarrow \cos 2\theta - \frac{1}{2}\sin 2\theta =$

$\Leftrightarrow 2\cos 2\theta = \sin 2\theta$

$\Leftrightarrow \tan 2\theta = 2$

(b) $\frac{2\tan\theta}{1-\tan^2\theta} = 2$

$\Leftrightarrow 2\tan^2\theta + 2\tan\theta - 1 = 0$

$\Leftrightarrow \tan\theta \frac{2 \pm \sqrt{12}}{4} = \frac{-1 \pm \sqrt{3}}{2}$

$\theta \in \left(0, \frac{\pi}{2}\right) \Rightarrow \tan\theta > 0$

$\therefore \tan\theta = \frac{-1+\sqrt{3}}{2}$ *[9 marks]*

15. (a) Add up the two formulae.

(b) Setting A – B = θ, A + B = 3θ gives A = 2θ, B = θ.

So, $\sin\theta + \sin 3\theta = 2\sin 2\theta\cos\theta = 2(2\sin\theta\cos\theta)\cos\theta = 4\sin\theta\cos^2\theta$ *[5 marks]*

16. (a) $\cos\theta = \dfrac{3-1+2}{\sqrt{11\times 6}} = 0.492 \;\therefore\; \theta = 60.5°$

(b) Solving equations for the second and third components gives $s = 3$, $t = 1$.

So, $1 + 1(3) = k + 3(1) \;\therefore\; k = 1$

(c) (i) $\begin{pmatrix} 2+1 \\ -6+1 \\ -3-1 \end{pmatrix} = \begin{pmatrix} 3 \\ -5 \\ -4 \end{pmatrix} = \boldsymbol{n}$

(ii) The plane contains the point $(4, -1, 3)$.

So the equation is: $3x - 5y - 4z = \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -5 \\ -4 \end{pmatrix} = 5$.

(d) Line through $(5, 1, 2)$ in the direction of the normal is $\boldsymbol{r} = \begin{pmatrix} 5 \\ 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -5 \\ -4 \end{pmatrix}$.

Intersects the plane when $3(5 + 3\lambda) - 5(1 - 5\lambda) - 4(2 - 4\lambda) = 5 \;\Rightarrow\; \lambda = \dfrac{1}{20}$.

The distance is $\left| \boldsymbol{r} - \begin{pmatrix} 5 \\ 1 \\ 2 \end{pmatrix} \right| = \left| \dfrac{1}{20} \begin{pmatrix} 3 \\ -5 \\ -4 \end{pmatrix} \right| = \dfrac{1}{20}\sqrt{9+25+16} = \dfrac{\sqrt{60}}{20} = \dfrac{\sqrt{15}}{5}$ *[15 marks]*