# **Revision answers: Calculus (Topic 6)**

**Coursebook chapters: 16–20**

**1.** *f* ′(*x*) = 3*ax*2 + 2*bx* + 4

*f* ″(*x*) = 6*ax* + 2*b*

So,

*f* ′(2) = 0 ⇒ 12*a* + 4*b* + 4 = 0 ⇒ 3*a* + *b* = −1

*f* ″(2) = 10 ⇒ 12*a* + 2*b* = 10 ⇒ 6*a* + *b* = 5

Solving simultaneously gives *a* = 2, *b* = −7 *[5 marks]*

**2. ** *[4 marks]*

**3.** (a) 

i.e.

(b) 

At *x* = 4,

∴ gradient of normal = −4

When *x* = 4, *y* = −2.

So, *y* – (−2) = −4(*x* – 4) ⇒ *y* = −4*x* + 14

(c) (i) At the intersection with the *x*-axis, *y* = 0: 0 = −4*x* + 14 ⇒ *x* =.

At the intersection with the *y*-axis, *x* = 0: *y* = 14.

So,, *Q*(0, 14)

(ii) Area  *[13 marks]*

**4.** From GDC, the intersection point is (0.7628, 0.9137) and the second curve has root 1.0327.

If *a* = 0.7628 and *b* = 1.0327, the area shown is.

Area = 0.5227 + 0.1207 = 0.643 (3SF) *[6 marks]*

**5.** (a) *y*′ = 12*x*3 – 24*x*2 + 12*x*

At stationary points *y*′ = 0

⇒ *x*3 – 2*x*2 + *x* = 0 ⇒ *x*(*x* – 1)2 = 0 ⇒ *x* = 0, 1

So stationary points are (0, −2) and (1, −1)

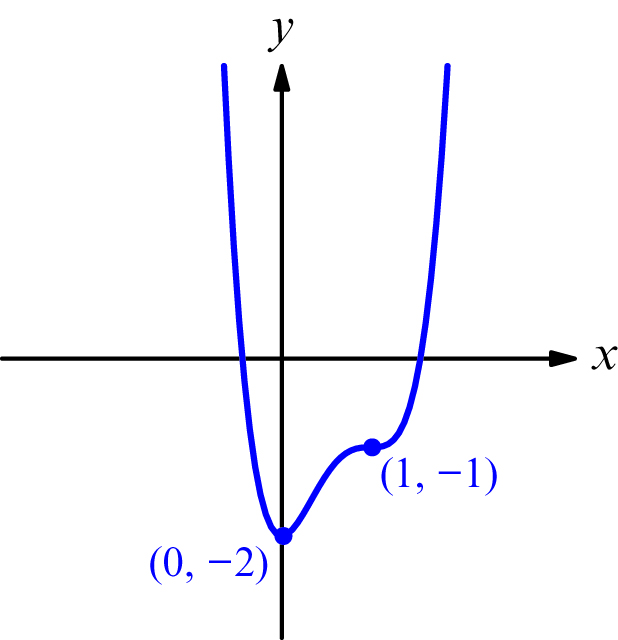
(b) *y*″ = 36*x*2 – 48*x* + 12

At *x* = 0, *y*″ = 12 > 0 ⇒ (0, −2) local minimum.

At *x* = 1, *y*″ = 0 so check *y*′ either side of *x* = 1.

When *x* =, *y*′ > 0 and when *x* =, *y*′ > 0 ⇒ (1, −1) (positive) point of inflexion.

(c)

 *[12 marks]*

**6.** Using the reverse chain rule or the substitution, *u* = *x*2 − 9: *[6 marks]*

**7.** Using the product rule for (a) and quotient rule for (b):

(a) *y*′ = 2e2*x* tan2 3*x* + e2*x* × 2(tan 3*x*) × 3 sec2 3*x*

= 2e2*x* tan2 3*x* + 6e2*x* tan 3*x* sec2 3*x*

= 2e2*x* tan 3*x*(tan 3*x* + 3 sec2 3*x*)

(b) 

  *[9 marks]*

**8.** (a) (i) *v* = d*t* = 3*t*2 – 22*t* + *c*

*v* = 35 when *t* = 0 ⇒ *c* = 35

∴ *v* = 3*t*2 – 22*t* + 35

(ii) 3*t*2 – 22*t* + 35 = 0 ⇒ (3*t* – 7)(*t* – 5) = 0 ⇒ *t* =, 5 seconds

(iii) For local max/min, *v*′ = 0, so 6*t* – 22 = 0 ⇒ *t* =.

However, at *t* = , *v* = , whereas at *t* = 0, *v* = 35.

∴ max speed = 35 ms−1

(b) We know that P has negative velocity for  < t < 5, so P is moving back towards O here.

So find the displacement for 0 < *t* <  and < t < 5 separately:

*s*1 = d*t* =  – 22*t* + 35 d*t* =  m

*s*2 = m

∴ *s* = ≈ 44 m *[11 marks]*

**9.** (a) (i) *y* = 3*x* ⇒ ln *y* = ln(3*x*) ⇒ ln *y* = *x* ln 3

Differentiating: *y*′ = ln 3 ⇒ *y*′ = *y* ln 3 = 3*x* ln 3

(ii) 

(b) *u* = 3*x* ⇒  = 3*x* ln 3 = *u* ln 3

When *x* = 0, *u* = 1 and when *x* = 1, *u* = 3.





 *[12 marks]*

**10.** 2*x* + 2*yy*′ − 3*y* – 3*xy*′ = 0 ⇒ *y*′ =

*y*′ = 0 ⇒ 3*y* – 2*x* = 0 ⇒ *y* =

Substituting into the original function:

*x*2 +  + 20 = 0 ⇒ *x*2 = 36 ⇒ *x* = ±6

So the stationary points are (6, 4) and (−6, −4). *[9 marks]*

**11.** (a) cos 2*x* = 1 – 2 sin2 *x* ⇒ sin2 *x* = (1 – cos 2*x*)



(b) *V* = *π* sin2 *x* d*x*





 *[12 marks]*

**12.** (a) Height of triangular face =

Area of triangular face of water of depth *h* and base *b*, *A* = *bh*

By similar triangles:

So, the volume of water at height *h*, *V* =.

(b) Capacity of tank = , so when quarter full *V* = .



Then, 

And 

So, when *h* =  *[7 marks]*

**13.** (a) *y* = ⇒ 

∴ sec2 *yy*′ =

(b) 



 *[9 marks]*

**14.** (a) 11 + 10*x* – *x*2 = 11 – (*x* – 5)2 + 25 = 36 – (*x* – 5)2

i.e. *a* = 36, *b* = 5

(b) 

⇒  *[8 marks]*