

**Self-assessment answers: 17 Basic integration and its applications**

$$1. \int \frac{\sqrt{x} - 3x^2}{4x^3} dx = \frac{1}{4} \int \left( \frac{\sqrt{x}}{x^3} - \frac{3x^2}{x^3} \right) dx$$

$$= \frac{1}{4} \int \left( x^{-\frac{5}{2}} - 3x^{-1} \right) dx$$

$$= -\frac{1}{6} x^{\frac{3}{2}} - \frac{3}{4} \ln x + c \quad [6 \text{ marks}]$$

$$2. \int_0^{\pi/3} 4 \cos x dx = [4 \sin x]_0^{\pi/3} = 2\sqrt{3} \quad [5 \text{ marks}]$$

$$3. \int_0^a e^x - x dx = 6.8$$

$$\Rightarrow \left[ e^x - \frac{x^2}{2} \right]_0^a = 6.8$$

$$\Rightarrow e^a - \frac{a^2}{2} - 1 = 6.8$$

$$\Rightarrow a = 2.36 \text{ (GDC)} \quad [6 \text{ marks}]$$

$$4. \quad (a) f(x) = x^2 - 6x + c$$

$$\text{Passes through } (2, 0) \Rightarrow 0 = 4 - 12 + c \Rightarrow c = 8$$

$$f(x) = x^2 - 6x + 8$$

$$(b) \text{ At } (2, 0), f'(x) = -2$$

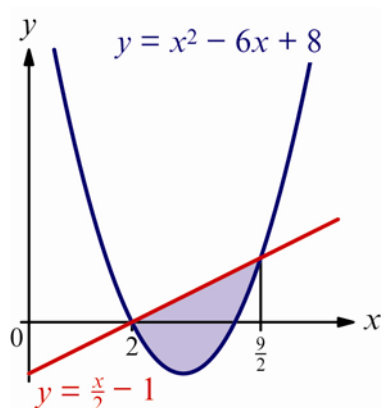
$$\text{So normal has gradient } \frac{1}{2} \text{ and therefore has equation } y = \frac{x}{2} - 1.$$

(c) Normal meets the curve when  $\frac{x}{2} - 1 = x^2 - 6x + 8$ .

$$\Rightarrow x^2 - \frac{13}{2}x + 9 = 0$$

$$\Rightarrow (x - 2)\left(x - \frac{9}{2}\right) = 0$$

Intersections are  $(2, 0)$  and  $\left(\frac{9}{2}, \frac{5}{4}\right)$ .



Area enclosed is given by  $\left| \int_2^{9/2} x^2 - \frac{13}{2}x + 9 dx \right|$

$$= \left| \left[ \frac{x^3}{3} - \frac{13}{4}x^2 + 9x \right]_2^{9/2} \right|$$

$$= \left| \frac{243}{8} - \frac{1053}{16} + \frac{81}{2} - \left( \frac{8}{3} - 13 + 18 \right) \right|$$

$$= \frac{125}{48} = 2.6$$

[13 marks]