# Prior learning self-assessment

Try the questions below then check your answers using the solutions at the end. If you've got anything wrong, do not recognise the words or symbols or feel uncomfortable with a topic it is recommended that you work through the corresponding Prior learning sheet on the CD-ROM.

## A. Arithmetic

**1.** Evaluate $\frac{1}{2}-\frac{3}{4}\times\frac{1}{3}+\frac{1}{8}$.

**2.** Evaluate $1\frac{2}{3}\div\left(1-\frac{5}{4}\right)$.

## B. Approximations and standard form

**1.** Write 1 345 987 to the nearest thousand.

**2.** Write 345 552 in standard form to three significant figures.

## C. Rational exponents

**1.** Evaluate $2\times 3^3$.

**2.** Evaluate $3\times 2^{-4}$, giving your answer as a fraction.

**3.** Evaluate $16^{\frac{3}{4}}$.

## D. Surds

**1.** Express $\left(2+\sqrt{8}\right)\left(3+\sqrt{2}\right)$ in the form $a+b\sqrt{2}$.

**2.** Express $\frac{1+\sqrt{3}}{5-\sqrt{3}}$ as a fraction with a rational denominator.

## E. Prime numbers and factors

**1.** Express 96 as a product of prime factors.

**2.** Find the greatest common divisor of 162 and 54.

## F. Ratio and proportion

**1.** Split 600 in the ratio 1 : 4 : 7.

**2.** If $a$ is proportional to the square root of $b$ and $a=24$ when $b=36$ find the value of $a$ when $b=100$.

## G. Sets

If $A=\{1,3,6,8\}$ and $B=\{\text{even numbers}\}$ and $U=\{\text{positive whole numbers less than } 10\}$

**1.** List the set $A \cap B$.

**2.** Find $n(A \cup B')$.

**3.** Is the following statement true or false: $\frac{3}{4} \in \mathbb{Q} \subset \mathbb{R}$?

## H. Probability

**1.** A die is thrown 100 times and a '6' occurs on 32 occasions. Based upon this information find the probability of getting a '6' when the die is next rolled.

**2.** A ten-sided die has one face showing a '1', two faces showing a '2', three faces showing a '3' and four faces showing a '4'. It is rolled once. What is the probability that the result is an even number?

## I. Interval notation and the modulus functions

**1.** Evaluate $|4-12|$.

**2.** Illustrate on a number line the regions $1 \le x < 4$ and $[2,5[$.

## J. Venn diagrams

**1.** Draw a Venn diagram to represent the relation between prime numbers, even numbers and square numbers with a universal set of all positive integers.

**2.** Draw a Venn diagram showing two overlapping regions $A$ and $B$. Shade in the area representing $A \cap B'$.

## K. Algebra of expressions

**1.** Expand and simplify $(x^2+3x+1)(3x+2)$.

**2.** True or false: $\sqrt{a^2+4b^2}=a+2b$?

## L. Solving equations and inequalities

**1.** Solve the equation $1+\frac{2x}{5}=3-4x$.

**2.** Solve the equation $5+\frac{3}{x}=4$.

3. Solve the inequality $3x-4>5x+2$.

## M. Working with formulae

1. Make $a$ the subject of the formula $b=\dfrac{3+a}{2-a}$.
2. If $a=3r^3+1$ and $b=a^2-1$, find $b$ in terms of $r$, giving your answer in simplified form.

## N. Factorisation

1. Factorise $14x^2y^3-21x^3y$.
2. Factorise $x^2-5x-6$.
3. Factorise $xy-y+1-x$.

## O. Algebraic fractions

1. Simplify the fraction:

$$\frac{x^2+6x+9}{x^2-9}$$

2. Write as a single fraction:

$$\frac{1}{x-1}-\frac{1}{x-2}$$

3. Write as a two level fraction:

$$\frac{2}{\left(\dfrac{7}{x}\right)}$$

## P. Cartesian geometry

1. What is the gradient of the line between the points $(3,7)$ and $(2,-3)$?
2. What is the distance between the points $(4,3)$ and $(6,8)$?

## Q. Simultaneous equations

**1.** Solve the simultaneous equations:

$x+3y=7$

$2x-y=2$

**2.** Solve the simultaneous equations:

$2x-y=1$

$xy=28$

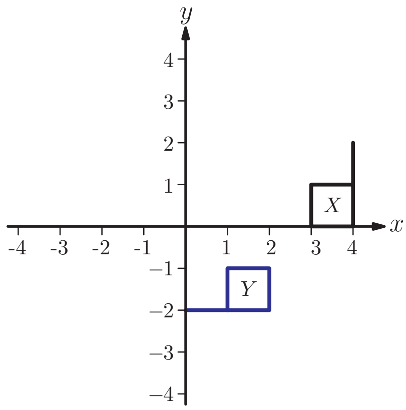
## R. Straight line graphs

**1.** Find the equation of the line connecting $(3,3)$ and $(5,7)$ in the form $y=mx+c$.

**2.** Find the equation of the line perpendicular to the line $x+2y=5$ through the point $(3,4)$.

## S. Geometric transformations

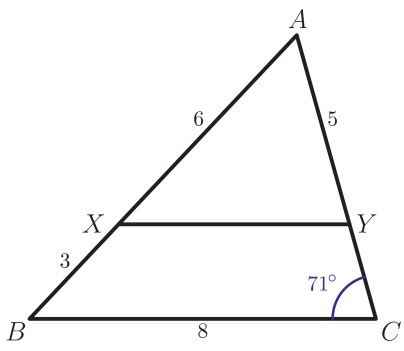
**1.** Describe fully the single transformation which changes shape $X$ into shape $Y$.



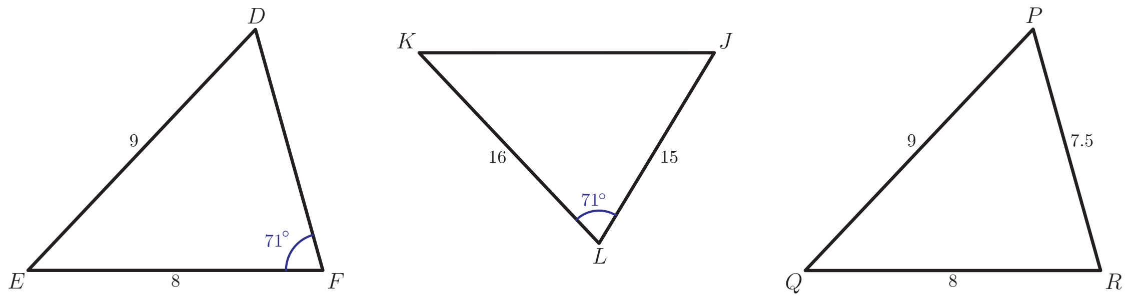

**2.** Sketch the shape $Y$ after it has been transformed by a rotation of $90°$ clockwise about the point $(1,1)$.

## T. Similar and congruent shapes

**1.** Find the length of side $XY$ in the triangle below:

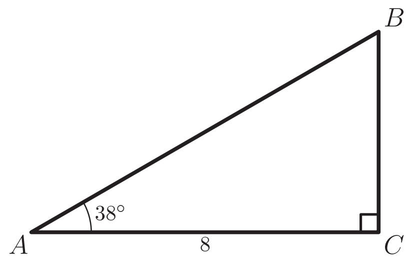


2. Decide which of the following triangles is necessarily congruent to triangle $ABC$ in the previous question. State the congruence condition.

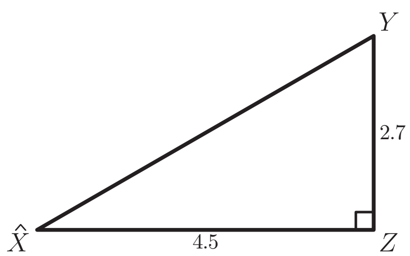



## U. Basic trigonometry

1. Find the length of the side $AB$ in the triangle below:



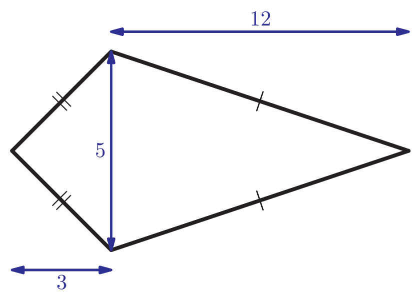

2. Find the size of angle $X$ in the triangle below:




3. A boat travels 6 km at a bearing of 030° followed by 4 km at a bearing of 120°. On what bearing and for what distance must it travel to return to its starting point?

## V. Area, perimeter and volume

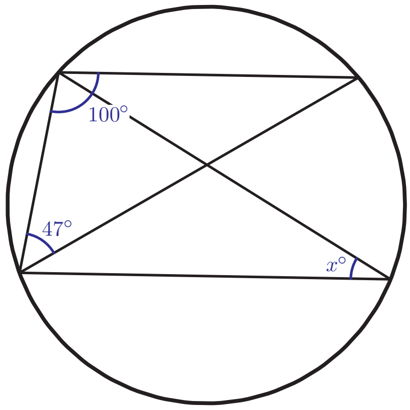
1. Find the area of the following shape:



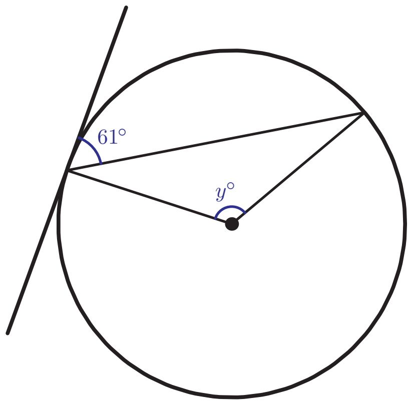

2. Find the volume of a cone with height 10 cm and radius 2 cm.

## W. Useful geometrical facts

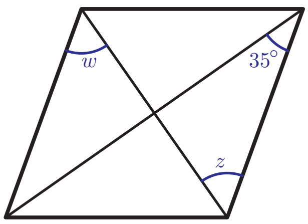
**1.** Find the angle marked $x$ below:




**2.** Find the angle marked $y$ below:

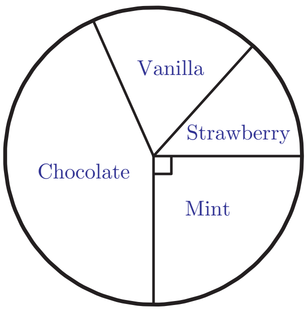



**3.** Find the angles marked $z$ and $w$ in this rhombus:




## X. Statistical diagrams

**1.** The following pie chart represents favourite ice cream flavours of 60 people. If 26 claimed chocolate as their favourite, and 8 preferred strawberry, how many people preferred vanilla?



2. Draw a frequency histogram and cumulative frequency diagram to represent the following data:

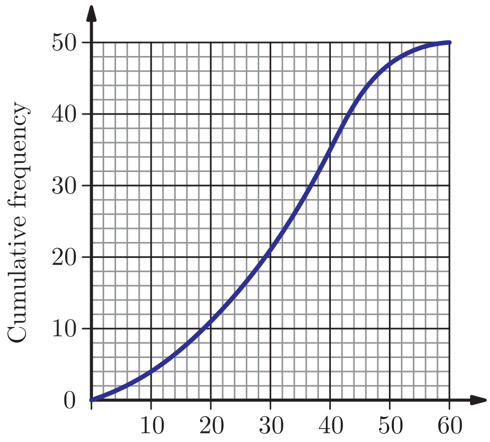
| $x$ | Frequency |
|---|---|
| 0 to 10 | 4 |
| 10 to 20 | 12 |
| 20 to 30 | 18 |
| 30 to 40 | 14 |
| 40 to 50 | 9 |

## Y. Statistical calculations

1. Find the mean, median and mode and range of the following data:

   15, 12, 4, 6, 12, 10

2. Estimate the median and inter-quartile range and 90th percentile from the following cumulative frequency diagram:




## Z. Quadratics

1. Write $2x^2 + 8x + 5$ in the form $a(x+h)^2 + k$.

2. Use the quadratic formula to solve the equation $x^2 + 6x - 1 = 0$.

3. Solve the quadratic inequality $x^2 > 5x$.

# Self-assessment answers

A **1.** $\frac{3}{8}$

**2.** $-6\frac{2}{3}$

B **1.** 1 346 000

**2.** $3.46\times10^5$

C **1.** 54

**2.** $\frac{3}{16}$

**3.** 8

D **1.** $10+8\sqrt{2}$

**2.** $\frac{4+3\sqrt{3}}{11}$

E **1.** $96=2^5\times3$

**2.** $\gcd(162,54)=54$

F **1.** $50:200:350$

**2.** $a=40$

G **1.** $\{6,8\}$

**2.** 7

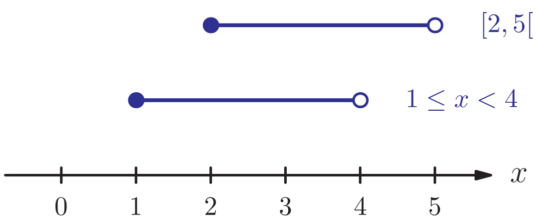
**3.** True

H **1.** $\frac{8}{25}$

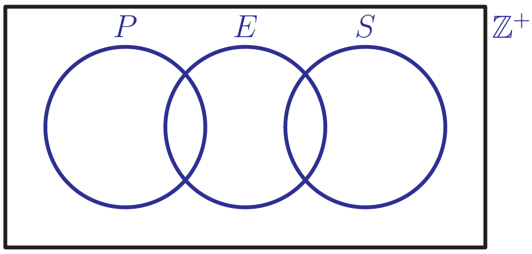
**2.** $\frac{3}{5}$

I **1.** 8

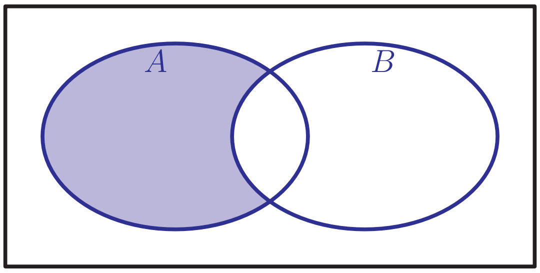
**2.**




J **1.**




**2.**




K **1.** $3x^3+11x^2+9x+2$

**2.** False (unless $a=0$ or $b=0$ )

L **1.** $x=\frac{5}{11}$

**2.** $x=-3$

**3.** $x<-3$

M **1.** $a=\frac{2b-3}{1+b}$

**2.** $b=9r^6+6r^3$

N **1.** $7x^2y(2y^2-3x)$

**2.** $(x-6)(x+1)$

**3.** $(x-1)(y-1)$

O **1.** $\frac{x+3}{x-3}$

**2.** $-\frac{1}{x^2-3x+2}$

**3.** $\frac{2x}{7}$

P **1.** 10

2. $\sqrt{29}$

**Q** 1. $x=\frac{13}{7}, y=\frac{12}{7}$

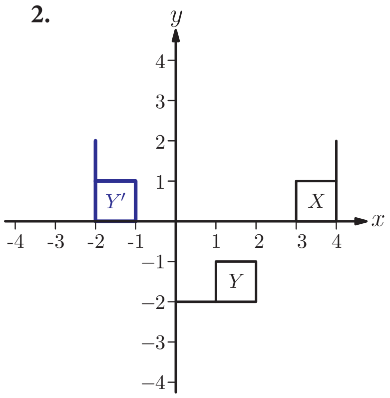
2. $(x,y)=(4,7)$ or $(-\frac{7}{2},-8)$

**R** 1. $y=2x-3$

2. $y=2x-2$

**S** 1. Reflection through $y=2-x$

2.




**T** 1. $5\frac{1}{3}$

2. *DEF* (unambiguous ASS), *PQR* (SSS). *JKL* is similar (SAS) but not congruent

**U** 1. 10.2

2. 31.0°

3. 7.21 km at 244°

**V** 1. 37.5

2. $\frac{40\pi}{3}=41.9\text{ cm}^2$

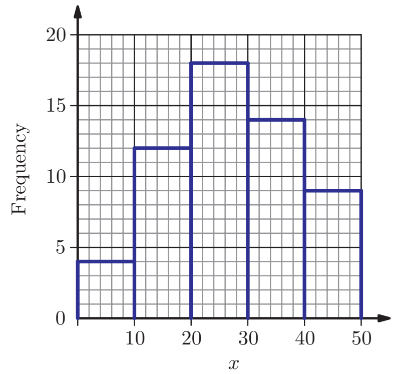
**W** 1. 33°

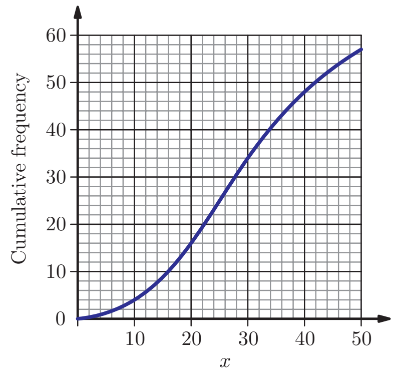
2. 122°

3. $w=z=55°$

**X** 1. 11

2.




**Y** 1. Mean = 9.83; median = 11; mode = 12; range = 11

2. Median 33, IQR 20, 90th percentile 47 (±1 is fine)

**Z** 1. $2(x+2)^2-3$

2. $x=\frac{-6\pm\sqrt{40}}{2}=-3\pm\sqrt{10}$

3. $x<0$ or $x>5$

## Section A Arithmetic

In the non-calculator section of the examination there might be times when you need to work with quite awkward numbers quickly and accurately. In particular you must be very familiar with rules for dealing with fractions.

It is also important to know the mathematical convention for the order in which we carry out operations:

1. Brackets
2. Exponents
3. Division & multiplication
4. Addition & subtraction

If several instances of operations in the same category occur in the same calculation we work through them from left to right.

### Worked example A.1

Evaluate without a calculator $\frac{1}{2}-\frac{1}{3}+\frac{1}{4}\div\frac{2}{5}$

The first operation we do is division. To divide fractions we flip the second one and multiply

$$\text{Expression} = \frac{1}{2}-\frac{1}{3}+\frac{1}{4}\times\frac{5}{2}$$

$$= \frac{1}{2}-\frac{1}{3}+\frac{5}{8}$$

For addition and subtraction we need a common denominator. An appropriate one here is 24

$$= \frac{12}{24}-\frac{8}{24}+\frac{15}{24}$$

$$= \frac{12-8+15}{24}$$

When there is both addition and subtraction we do them in the order they occur

$$= \frac{4+15}{24}$$

$$= \frac{19}{24}$$

### Exercise A

Evaluate the following without a calculator. Simplify fractions where possible.

**1.** (a) $7-3^2$ (b) $5-3(8-1)$ (c) $-5^2$

(d) $(-5)^2$ (e) $\frac{65}{5}+98$ (f) $12\div(6-2)$

**2.** (a) $\frac{5}{12}+\frac{3}{4}$ (b) $\frac{2}{7}+\frac{9}{5}$ (c) $\frac{15}{23}-\frac{1}{2}$

(d) $\frac{5}{7}-1$ (e) $23-\frac{14}{15}+\frac{2}{5}$ (f) $\frac{45}{22}+\frac{5}{11}-\frac{1}{44}$

**3.** (a) $\frac{3}{5}\times\frac{7}{6}$ (b) $\frac{9}{10}\times\frac{5}{6}$ (c) $\frac{12}{5}\div\frac{12}{7}$

(d) $\frac{23}{6}\div\frac{5}{12}$ (e) $\frac{9}{12}\div\frac{6}{12}\times\frac{2}{3}$ (f) $\frac{1}{5}\times\frac{2}{5}\div\frac{4}{5}$

**4.** (a) $3\frac{2}{3}+\frac{1}{9}$ (b) $4-2\frac{1}{3}$ (c) $1\frac{1}{2}\times 2\frac{1}{3}$

(d) $3\frac{2}{3}\div 3$ (e) $1\frac{4}{7}+\frac{3}{4}\times\frac{1}{7}$ (f) $1\frac{3}{4}\div\left(2-\frac{5}{8}\right)$

# Section B Approximations and standard form

You must be able to round to:

- A given order of magnitude; for example, the nearest thousand
- A set number of decimal places
- A set number of significant figures

In all situations you look at the digit immediately to the right of the one you are rounding to. If it is a five or more, you add one to the last digit you are rounding to. If it is four or less you leave the last digit the same. You then fill out the required number of zeros to keep the number at the same order of magnitude.

The most complicated type of rounding to deal with is significant figures. These are counted from the first
non-zero digit (from left to right).

> **EXAM HINT**
>
> In International Baccalaureate® exams, you should always give your answers to three significant figures, unless otherwise stated. Otherwise you will lose one mark per paper.

## Worked example B.1

Round the following to three significant figures:

(a) 0.000 310 75

(b) 134 512

(c) 1.074 6

| | |
|---|---|
| Ignore the initial zeros to find the first three significant figures | (a) **0.000310**754 |
| The next number along is a seven, so we round up | =0.000311 (3SF) |
| The fourth significant figure is 5, so round up | (b) **134**512 |
| Fill in zeros at the end to keep the same order of magnitude | =135000 (3SF) |
| Zero in the middle counts as a significant figure | (c) **1.07**46 |
| The next number along is 4, so round down | =1.07 (3SF) |

Standard form is a convenient way of writing very large or very small numbers. We write it in the form $a \times 10^n$, where $a$ is between 1 and 10 and $n$ is a whole number.

**Worked example B.2**

Write 134 000 in standard form

| | |
|---|---|
| Write the initial digits putting a decimal point after the first digit | 1.34 |
| Look at what power of ten the first digit represents | $10^5$ |
| Multiply these numbers together | $1.34 \times 10^5$ |

## Exercise B

**1.** Round the following numbers to the degree of accuracy given.

(a) 34 562 (3SF) (b) 23.544 (2DP) (c) 487.3 (Nearest 1000)

(d) 99 999 (3SF) (e) 789.645 (1DP) (f) 12.984 (Nearest tenth)

**2.** Write the following numbers in standard form.

(a) 1 598 000 000 (b) 0.0000 56 4 (c) 2.356

(d) 23.56 (e) 0.2356 (f) 18976500

**3.** Write the following numbers in full decimal form.

(a) $6.5 \times 10^7$ (b) $7.12 \times 10^{-3}$ (c) $8.54 \times 10^1$

(d) $8.88 \times 10^0$ (e) $1.152 \times 10^{-7}$ (f) $2.365\,4 \times 10^8$

## Section C Rational exponents

In exponent form, the first number is called the base and the second is called the power or the exponent.

$$3^{x+2y} = \underbrace{3}_{\text{base}}^{\overbrace{x+2y}^{\text{power}}}$$

You must be particularly careful when the base is negative; it is very easy to get your signs wrong.

**Worked example C.1**

Evaluate $(-4)^3$

Write the exponent form as repeated multiplication

$(-4)\times(-4)\times(-4)$

Evaluate in parts

$=16\times(-4)$

$=-64$

If the exponent is increased by one, the value of the expression is multiplied by the base. If the exponent is decreased by one, the value of the expression is divided by the base. It follows therefore that the value of $a^0$ must (to be consistent) be the value of $a^1$ divided by $a$. So, $a^0 = a^1 \div a = 1$.

KEY POINT C.1

$a^0 = 1$

Further, if we continue dividing by the value of the base, we enter the realm of negative exponents:

$a^{-1} = a^0 \div a = \dfrac{1}{a}$ For example: $3^{-1} = \dfrac{1}{3}$

$a^{-2} = a^{-1} \div a = \dfrac{1}{a^2}$ For example: $3^{-2} = \dfrac{1}{9}$

Generalising this gives:

KEY POINT C.2

$$a^{-n} = \frac{1}{a^n}$$

There is also a rule for dealing with rational exponents. This will be justified in chapter 2 of the coursebook.

KEY POINT C.3

$$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$$

## Worked example C.2

Evaluate $64^{\frac{2}{3}}$

Use $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$

$64^{\frac{2}{3}} = \left(64^{\frac{1}{3}}\right)^2 = (4)^2 = 16$

## Exercise C

1. Are the following statements true or false?

(a) $x^2$ is always greater than $x$ (b) $3\times 5^7 = 15^7$ (c) $6^{14} = 6\times 6^{13}$

(d) $1^{99} = 1$ (e) $0^{40} = 1$ (f) $9^7 = 7^9$

(g) $2^4 + 2^4 = 2^5$ (h) $3^4 + 3^4 = 3^5$

2. Evaluate:

(a) $2^5$ (b) $7^2$ (c) $6^3$

(d) $3^3$ (e) $3^2 + 2^3$ (f) $\dfrac{5^3 - 5^2}{2^2}$

(g) $2\times 3^2$ (h) $5\times 2^3$ (i) $\left(\dfrac{2}{3}\right)^3$

(j) $\left(-\dfrac{1}{4}\right)^2$ (k) $\left(-\dfrac{5}{3}\right)^3$

3. Evaluate, leaving your answer as a fraction:

(a) $3^{-1}$ (b) $6^{-1}$ (c) $7^{-2}$

(d) $4^{-3}$ (e) $3\times 2^{-2}$ (f) $6\times 5^{-1}$

4. Evaluate the following:

(a) $4^{\frac{1}{2}}$ (b) $27^{\frac{1}{3}}$ (c) $32^{\frac{3}{5}}$

(d) $1^{\frac{1}{3}}$ (e) $25^{-\frac{1}{2}}$

## Section D Surds

A surd (also called a root or a radical) is any number of the form $a+b\sqrt{c}$ where *a*, *b*, and *c* are fractions or whole numbers.

The most important rule of surds deals with their product:

$$\sqrt{ab}=\sqrt{a}\sqrt{b}$$

There is no equivalent for the sum. In general:

$$\sqrt{a+b}\neq\sqrt{a}+\sqrt{b}$$

We can collect square roots of the same number together, so for example $3\sqrt{2}+5\sqrt{2}=8\sqrt{2}$.

One very useful tool for simplifying square roots is to take out any square factors of the number being square rooted. For example:

$$\sqrt{18}=\sqrt{9\times 2}=\sqrt{9}\times\sqrt{2}=3\sqrt{2}\,.$$

### Worked example D.1

Write $\left(\sqrt{6}+\sqrt{2}\right)^2$ in the form $a+b\sqrt{3}$

Treat the expression as two brackets

$$\left(\sqrt{6}+\sqrt{2}\right)\left(\sqrt{6}+\sqrt{2}\right)$$

$\sqrt{2}\times\sqrt{2}=2$

$$=6+\sqrt{6}\sqrt{2}+\sqrt{2}\sqrt{6}+2$$

$\sqrt{2}\times\sqrt{6}=\sqrt{12}$

$$=8+2\sqrt{12}$$

Then put it into the required form

$$=8+2\sqrt{4}\sqrt{3}$$

$$=8+4\sqrt{3}$$

Another important simplification is called rationalising the denominator.

In most situations, the denominator of a fraction should be an integer. When presented with a fraction with a surd in the denominator we must change the form of the fraction.

If the denominator is simply a square root then you multiply top and bottom of the fraction by this square root. If it is a surd expression then you have to multiply top and bottom by the conjugate of the surd – this is the same surd with the sign in front of the square root changed.

### Worked example D.2

Rationalise the denominator of $\dfrac{2}{3+\sqrt{2}}$

Multiply top and bottom by the conjugate of the bottom

$$=\frac{2}{3+\sqrt{2}}\times\frac{3-\sqrt{2}}{3-\sqrt{2}}$$

$$=\frac{6-2\sqrt{2}}{9+3\sqrt{2}-3\sqrt{2}-2}$$

$$=\frac{6-2\sqrt{2}}{7}$$

## Exercise D

**1.** Write in the form $k\sqrt{5}$, where $k$ is a whole number:

(a) $7\sqrt{5}-2\sqrt{5}$ (b) $\sqrt{5}+9\sqrt{5}-3\sqrt{5}$ (c) $\sqrt{20}+\sqrt{45}$

(d) $\sqrt{5}-2(\sqrt{125}-\sqrt{20})$ (e) $3\sqrt{80}-5\sqrt{20}$ (f) $\sqrt{80}+7\sqrt{5}$

**2.** Write in the form $\sqrt{a}$, where $a$ is a whole number:

(a) $4\sqrt{2}$ (b) $7\sqrt{3}$ (c) $\sqrt{7}+2\sqrt{7}$

(d) $\sqrt{3}+\sqrt{75}$ (e) $\sqrt{2}+\sqrt{8}$ (f) $\sqrt{32}+\sqrt{8}$

**3.** Write in the form $a+b\sqrt{3}$:

(a) $2\left(3-\sqrt{3}\right)-3\left(1-\sqrt{3}\right)$ (b) $\left(1+\sqrt{3}\right)-\left(1-\sqrt{3}\right)$ (c) $\left(1+\sqrt{3}\right)\left(2+\sqrt{3}\right)$

(d) $\left(1-\sqrt{3}\right)\left(1+2\sqrt{3}\right)$ (e) $\left(4-2\sqrt{3}\right)^2$ (f) $\left(\sqrt{15}-\sqrt{5}\right)^2$

**4.** Rationalise the denominators, simplifying your answer where possible:

(a) $\dfrac{1}{\sqrt{7}}$ (b) $\dfrac{3-\sqrt{6}}{\sqrt{6}}$ (c) $\dfrac{3}{4-\sqrt{3}}$

(d) $\dfrac{1}{\sqrt{2}-1}$ (e) $\dfrac{1+\sqrt{5}}{1+\sqrt{7}}$ (f) $\dfrac{3+\sqrt{2}}{3-\sqrt{2}}$

**5.** Write in the form $a+b\sqrt{2}$:

(a) $\dfrac{1}{\sqrt{2}}+\sqrt{2}$ (b) $\dfrac{3}{\sqrt{2}}+\dfrac{5}{2+\sqrt{2}}$ (c) $\dfrac{1}{1+\sqrt{2}}-\dfrac{1}{1-\sqrt{2}}$

## Section E Prime numbers and factors

A prime number is any number that is exactly divisible only by one and itself. The first few prime numbers are 2, 3, 5, 7, 11, 13, 17, 19.

**EXAM HINT**

Notice that 1 is NOT a prime number

A factor of $x$ is a positive integer that divides $x$ exactly. A multiple of $x$ is an integer that $x$ divides exactly.

If we have two numbers $x$ and $y$, we can find the highest factor common to both; this is called the greatest common divisor (gcd). (You might have seen this before as the highest common factor (HCF).)

We can also find the lowest multiple common to both of them; this is called the least common multiple (lcm). (Also called the lowest common multiple (LCM).)

The lcm is harder to work out than the gcd unless you remember a very useful formula:

$$\operatorname{lcm}(x, y) = \frac{xy}{\gcd(x, y)}$$

**Worked example E.1**

Find the prime factorisation of 36.

Keep factorising until all factors are prime

$$\begin{aligned} 36 &= 18 \times 2 \\ &= 9 \times 2 \times 2 \\ &= 3 \times 3 \times 2 \times 2 \end{aligned}$$

**Worked example E.2**

What are the greatest common divisor and least common multiple of 32 and 56?

Factors of 32 are 1, 2, 4, 8, 16, 32

Factors of 56 are 1, 2, 4, 7, 8, 14, 28

So the greatest common divisor is 8

Use the formula linking gcd and lcm

Least common multiple

$$\begin{aligned} &= \frac{32 \times 56}{8} \\ &= 224 \end{aligned}$$

## Exercise E

**1.** Write as a product of prime factors:

(a) 72 (b) 50 (c) 32

(d) 27 (e) 29 (f) 1001

**2.** Find the gcd and lcm of:

(a) 12 and 25 (b) 17 and 16 (c) 18 and 27

(d) 42 and 56 (e) 26 and 39 (f) 14 and 32

(g) 50 and 25

**EXAM HINT**

You may be able to find gcd and lcm using your calculator.

# Section F Ratio and proportion

Ratio is a way of comparing two or more groups. For example, if something is split in a ratio $3:5$ there are three parts of the first thing to every five parts of the second thing.

The way to do this is to split the original quantity into eight equal parts (the sum of 3 and 5).

Proportion is another word for fraction; it is a way of comparing one group to the whole quantity. So the proportion of vowels in the alphabet is $\frac{5}{26}$.

If two variables are 'in proportion' then if you divide one by the other the result is always the same. That is, if $y$ is proportional to $x$ then $\frac{y}{x} = k$ is a constant. We usually just rewrite this as $y = kx$.

### Worked example F.1

A line is 7 cm long. If the line is split by a mark in the ratio $3:7$, what is the length of the shorter segment of the line?

Divide the total into $3 + 7 = 10$ parts

Total of 10 parts, so each part is 0.7 cm. The shorter segment of the line is $3 \times 0.7\text{ cm} = 2.1\text{cm}$

### Worked example F.2

If $a$ is proportional to $b$ and when $a = 6$, $b = 10$, what is the value of $a$ when $b = 34$?

If $a$ is proportional to b then $a = kb$

Use the given values to find $k$

When $a = 6$, $b = 10$ so

$6 = 10k$

$k = 0.6$

When $b = 34$:

$a = 0.6 \times 34$

$= 20.4$

## Exercise F

**1.** Split in the ratios given:

(a) 100 in the ratio 2:3 (b) 800 in the ratio 17:8

(c) 200 in the ratio 9:7 (d) 490 in the ratio 1:2:4

(e) 60 in the ratio 1:7:16 (f) 19 in the ratio 1:2:2

**2.** (a) If $a$ is proportional to $b$ and when $a=3, b=6$, find a formula for $a$ in terms of $b$.

(b) If $r$ is proportional to $s$ and when $r=20, s=15$, find a formula for $r$ in terms of $s$.

(c) If $x$ is proportional to $y$ and when $x=4, y=10$, find $x$ when $y=12$.

(d) If $x$ is proportional to $y$ and when $x=10, y=7$, find y when $x=25$.

(e) If $p$ is proportional to $q^2$ and when $p=5, q=10$, find a formula for $p$ in terms of $q$.

(f) If $r$ is proportional to $\sqrt{s}$ and when $r=1, s=4$, find $r$ when $s=16$.

# Section G Sets

A set is a collection of objects. This can be an infinitely large collection, but there must be a rule to decide whether or not any object is in the set. If we list the objects in a set we write the list in curly brackets, e.g. $A = \{1,3,5\}$.

There are several symbols which are used when we describe sets:

| | |
|---|---|
| $x \in A$ | $x$ is a member of the set $A$. |
| $x \notin A$ | $x$ is not a member of the set $A$. |
| $n(A)$ | The number of members of set $A$. |
| $U$ | The universal set; all objects of interest. If this is not defined or implicit from context, we usually take it to be all positive whole numbers. (In some courses, this is represented by $\varepsilon$.) |
| $A'$ | The complement of $A$; everything that is in $U$ but not in $A$. |
| $\varnothing$ | The empty set; a set containing no objects. |
| $A \cup B$ | '$A$ union $B$'; a set made up of everything that occurs in either $A$ or $B$ or both. |
| $A \cap B$ | '$A$ intersection $B$'; a set made up of everything that occurs in both $A$ and $B$. |
| $B \subset A$ | $B$ is a proper subset of $A$. |
| $B \subseteq A$ | $B$ is a subset of $A$ but also equal to $A$. |
| $B \not\subset A$ | $B$ is not a proper subset of $A$. |
| $B \not\subseteq A$ | $B$ is not a subset of $A$. |

**Worked example G.1**

If $F = \{5,6,7,8,9\}$ and $D = \{\text{positive odd numbers less than 10}\}$, list $F \cap D$. Find $n(F \cap D)$.

Writing $D$ as a list makes things easier

$D = \{1,3,5,7,9\}$

What objects are in both $F$ and $D$?

$F \cap D = \{5,7,9\}$

How big is this set?

$n(F \cap D) = 3$

There are many different types of number and it is very important that you know some of the labels applied to them:

| | | |
|---|---|---|
| $\mathbb{N}$ | Natural numbers: | $\{0, 1, 2, 3, 4\ldots\}$ |
| $\mathbb{Z}$ | Integers: | $\{\ldots -3, -2, -1, 0, 1, 2, 3\ldots\}$ |
| $\mathbb{Z}^+$ | Positive integers | $\{1, 2, 3, 4\ldots\}$ |
| $\mathbb{Q}$ | Rational numbers | $\{\ldots -\frac{3}{5}, 0, 5.3, 6, \frac{43984}{865}\ldots\}$ |
| $\mathbb{Q}'$ | Irrational numbers | $\{\ldots -\sqrt{2}, 1-\sqrt{3}, \pi\ldots\}$ |
| $\mathbb{R}$ | Real numbers | $\{\ldots -\sqrt{2}, 0, \pi, 5.3, 6, \frac{43984}{865}\ldots\}$ |

A rational number is any number which can be written as a fraction of two integers. Irrational numbers cannot be written as a fraction of two integers. You are already familiar with some irrational numbers: surds and anything involving $\pi$. The real numbers comprise all numbers you have met so far. If no other indication is given, assume that you are working with real numbers.

You may wonder what other types of numbers exist. Infinity does not fit into any of the sets above. Nor does $\sqrt{-1}$ which is called an imaginary number. We can treat coordinates as if they are a different type of number too.

## Exercise G

**1.** If $\mathbb{U} = \{\text{Positive whole numbers less than } 20\}$, $A = \{5, 7, 15, 17\}$, $B = \{\text{even numbers}\}$, $C = \{\text{multiples of } 5\}$ and $D = \{\text{Prime numbers}\}$

(a) List $A \cap C$
(b) List $A' \cap C$
(c) List $A \cup D$
(d) List $(B \cup D)'$
(e) Find $n(A \cap C)$
(f) Find $n(B \cup C)$

**2.** Tick the boxes to indicate which numbers are in which sets. The first one has been done for you.

| | Number | $\mathbb{N}$ | $\mathbb{Z}$ | $\mathbb{Q}$ | $\mathbb{R}$ |
|---|---|---|---|---|---|
| **(a)** | $0$ | ✓ | ✓ | ✓ | ✓ |
| **(b)** | $1$ | | | | |
| **(c)** | $\frac{1}{3}$ | | | | |
| **(d)** | $-6$ | | | | |
| **(e)** | $-1.5$ | | | | |
| **(f)** | $0.\dot{3}$ | | | | |
| **(g)** | $\sqrt{2}$ | | | | |
| **(h)** | $\sqrt{25}$ | | | | |
| **(i)** | $\pi + 2$ | | | | |

# Section H Probability

The probability of an event '$A$' occurring is denoted by P($A$).

This is a measure of how likely an event is to occur, and it can be estimated by looking at how often the event has occurred.

KEY POINT H.1

From observation, we can estimate P($A$) as:

$$\mathrm{P}(A) = \frac{\text{Number of times } A \text{ occurs}}{\text{Number of times } A \text{ could have occurred}}$$

Notice from this definition P($A$) will be between 0 and 1 inclusive.

**Worked example H.1**

A six-sided die is rolled 200 times and a '6' occurs 40 times. Estimate the probability of a '6' occuring on the next roll.

Apply the formula

$$\mathrm{P}(6) = \frac{40}{200} = \frac{1}{5}$$

We can estimate the probability empirically by doing an experiment repeatedly, but in some situations it is also possible to predict the probability *before* the experiment. For example, when you toss a fair coin there are two possible outcomes: heads or tails. Since both are equally likely, the probability of each must be one half. This leads to a theoretical definition of probability.

KEY POINT H.2

$$\mathrm{P}(A) = \frac{\text{Number of } A \text{ outcomes}}{\text{Total number of (equally likely) outcomes}}$$

Again we have $0 \le \mathrm{P}(A) \le 1$.

We can now interpret P($A$):

If P($A$) = 0, then the event $A$ is impossible.

If $\mathrm{P}(A) = 1$ then event $A$ is certain.

As $\mathrm{P}(A)$ rises the likelihood of $A$ occurring increases.

There are two possible outcomes when you enter a lottery: either you win or you do not win. But this does not mean that the probability of winning is one half, since there is no reason to believe that both outcomes are equally likely. Many mistakes in probability arise from this type of error.

## Worked example H.2

What is the probability of getting a prime number when you roll a six-sided die?

List all the possible outcomes and make sure they are all equally likely

Possible outcomes are 1, 2, 3, 4, 5, 6

Identify how many of them are prime

2, 3 and 5 are prime, which is 3 outcomes out of a possible 6

Write this as a probability

So probability is $\frac{3}{6}$ $(=\frac{1}{2})$

## Exercise H

**1.** In this question, give answers as decimals and round answers to three significant figures where appropriate.

(a) (i) 600 boxes of cereal are opened and 400 contain plastic toys. Estimate the probability that a randomly opened box contains a plastic toy.

(ii) 2000 people are surveyed and 300 say that they will vote for the Green party in a forthcoming election. Estimate the probability that a randomly chosen voter will vote for the Green party.

(b) (i) In a boardgame some cards are drawn; 4 of the cards give positive effects and 3 of them give negative effects. Estimate the probability that a randomly chosen card gives a positive effect.

(ii) While practising basketball free throws, Brian scores 12 and misses 18. Estimate the probability that Brian scores in his next attempt.

(c) (i) From a large bag of marbles, several are drawn: 3 are red, 6 are blue and 5 are green. Estimate the probability that the next randomly drawn marble is blue.

(ii) Yuna picked 20 DVDs randomly from her collection: 12 were comedy, 6 were action and the remainder were documentaries. Estimate the probability that another randomly chosen DVD from her collection is a documentary.

**2.** (a) The probability of getting a Heart when a card is drawn from a normal pack of cards is $\frac{1}{4}$. 20 cards are chosen with replacement. How many Hearts would you expect to draw?

(b) A cancer drug shows a positive effect in 15% of cases within one year. If 500 cancer patients are put on a trial of the drug, how many would you expect to show a positive effect after one year?

3. In a standard pack of 52 playing cards there are 4 different suits (red Hearts, red Diamonds, black Clubs and black Spades). In each suit there are number cards from 2 to 10, then four 'picture cards'; Jack, Queen, King and Ace. Giving your answer as a fraction, find the probability that a randomly chosen card is:

(a) (i) Red (ii) A Spade

(b) (i) A Jack (ii) A picture card

(c) (i) A black number card (ii) A Club picture card

4. A bag contains three different kinds of marble: six are red, four are blue and five are yellow. One marble is taken from the bag. Calculate the probability that it is:

(a) (i) Red (ii) Yellow

(b) (i) Not blue (ii) Not red

(c) (i) Green (ii) Not green

# Section I Interval notation and the modulus function

The set of all real numbers between two given values is called an **interval**.

Intervals can be described using inequalities and illustrated on number lines.

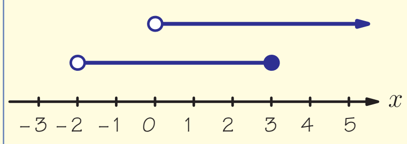
The five inequalities you need to know are:

- $<$ Less than
- $>$ Greater than
- $\leq$ Less than or equal to
- $\geq$ Greater than or equal to
- $\neq$ Not equal to

We represent these on number lines by drawing lines above the axis representing the numbers that are included in the inequality. There is a convention that when the line stops and the end point is included we use a filled in dot, and when the line stops but the end point is not included we use an unfilled dot. To indicate a line continuing indefinitely, we end it in an arrow.

**Worked example I.1**

Represent on a number line the inequalities $-2 < x \leq 3$ and $x > 0$ and list the integers satisfying both inequalities.




So the whole numbers satisfying both are 1, 2, 3

Intervals can also be described using **interval notation** instead of inequalities.

| Inequality | Interval |
|---|---|
| $2 \leq x \leq 5$ | $[2,5]$ |
| $2 < x < 5$ | $]2,5[$ |
| $-5 \leq x < 1$ | $[-5,1[$ |
| $x > 0$ | $]0,\infty[$ |
| $x \leq 2$ | $]-\infty,2]$ |

Note that a bracket facing inwards means that the end point is included, and a bracket facing outwards means that the end point is not included in the interval.

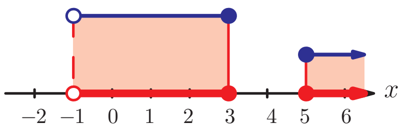
Many mathematical cultures use round brackets to show that an endpoint is not included, so $]2,5]$ would be written as $(2,5]$. However, in this book we will always use the International Baccalaureate® notation as described in the table.

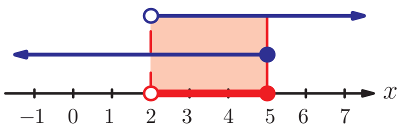
The symbol $\infty$ is called **infinity** and represents the fact that the interval does not have an upper limit. It can never be included as an end point, so is always given with an outward facing bracket.

The concept of infinity raises many mathematical and philosophical questions. You may have heard that there are 'different types of infinity'. One important thing to remember is that $\infty$ is *not* a real number. In the context used here, it is simply a symbol indicating that all numbers above a certain value are included in the interval.

Since an interval is a set of numbers, we can use the set notation symbol $\in$ to indicate that a number lies inside an interval. So $x \in ]-2,3]$ means the same as $-2 < x \le 3$.

We can also use the set concepts of union and intersection to combine intervals.




In the first diagram, $x$ can lie in either of the two separate intervals: $-1 < x \le 3$ ***or*** $x \ge 5$. So it lies in their union: $x \in ]-1,3] \cup [5,\infty[$.

In the second diagram, $x$ needs to be in both intervals: $x > 2$ ***and*** $x \le 5$. So it lies in their intersection: $x \in ]2,\infty[ \cap ]-\infty,5]$. In this case we can use a single interval: $2 < x \le 5$, or $x \in ]2,5]$.

The modulus function, $|x|$, does nothing to positive numbers but strips away the negative sign from negative numbers. For example $|5.2| = 5.2$ and $|-9| = 9$. Some intervals can be conveniently described using the modulus function. For example, $|x| < 5$ represents the interval $-5 < x < 5$ and $|x| > 5$ represents the union of two intervals: $x < -5$ ***or*** $x > 5$.

## Exercise I

**1.** Evaluate:

(a) $|-6|$ (b) $|-14|$ (c) $|8|$

(d) $|16|$ (e) $|-0.6|$ (f) $\left|\dfrac{1}{5}\right|$

(g) $|9-12|$ (h) $\left|\dfrac{1}{-5}\right|$

**2.** Represent the following inequalities on a number line. List the whole numbers satisfying all the given inequalities.

(a) $1 < x \le 5$ (b) $1 \ge x \ge -2$ (c) $x < 4$ and $x \ge -2$

(d) $x \le 0$ and $x \ge -5$ (e) $x \le 3$ and $0 < x < 3$ (f) $-2 \le x \le 4$

(g) $-2 \le x \le 5$ and $2 < x < 5$ (h) $-3 < x \le 2$ and $0 < x \le 3$

**3.** Write the following statements using inequalities:

(a) $x \in [1,2]$ (b) $x \in ]-\infty, 4]$ (c) $x \in [1,6[$

(d) $x \in ]-3, \infty[$ (e) $x \in ]-3, 3[$ (f) $x \in [5, \infty[$

**4.** Write the following statements using interval and set notation. Write them as a single interval where possible.

(a) $1 \le x \le 3$ and $x > 2$ (b) $1 \le x < 5$ or $x \ge 7$ (c) $x \le -2$ or $x \ge 2$

(d) $x \le -1$ and $1 < x < 3$

Do any of the statements represent the empty set?

**5.** Write the following inequalities using the modulus function:

(a) $-2 < x < 2$ (b) $-5 \le x \le 5$ (c) $4 \ge x \ge -4$

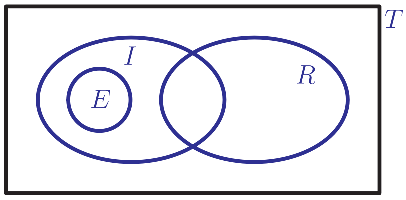
(d) $x < -2$ or $x > 2$ (e) $x \ge 10$ or $x \le -10$

# Section J Venn diagrams

A Venn diagram is a way of using regions to represent sets. If they overlap it means that there are some items in both sets.

**Worked example J.1**

Draw a Venn diagram representing the relationship between isosceles triangles ($I$), right-angled triangles ($R$) and equilateral triangles ($E$).




Notice that the box around the circles represents everything else that is relevant; the universal set. In this case it might be all triangles ($T$).

Interpreting the Venn diagram we could say that some isosceles triangles are right-angled, all equilateral triangles are isosceles, and some triangles are neither isosceles nor right-angled.

## Exercise J

**1.** Draw Venn diagrams to represent the following relationships

(a) Squares, rectangles, parallelograms (Universal set: all quadrilaterals).

(b) Even numbers, prime numbers, square numbers (Universal set: all positive whole numbers).

(c) Quadrilaterals, hexagons, rectangles (Universal set: all polygons).

(d) Higher level mathematics students, standard level mathematics students, higher level physics students (Universal set: all IB students).

(e) $\mathbb{Z}, \mathbb{Q}, \mathbb{Q}'$ (Universal set: $\mathbb{R}$).

**2.** Draw separate Venn diagrams showing two overlapping regions, $A$ and $B$, and shade the following areas:

(a) $A \cup B$ (b) $A \cap B$ (c) $A'$

(d) $A' \cap B$ (e) $A' \cap B'$ (f) $A' \cup B'$

# Section K Algebra of expressions

Can you write down 'the rules of algebra'? Do you have to be able to describe something to do it?

The rules of algebra are difficult to describe, so we shall just outline some of the most important things to remember.

- Rules of arithmetic:

  All the rules of arithmetic apply equally to algebra. In particular, the same order of operations applies.
- Combining terms:

  If we are adding terms together, we can group together terms which are alike. This means that, for example, $2x$ and $5x$ can be collected to form $7x$, but $x$ and $x^2$ cannot be collected together.
- Dealing with brackets:

  If we multiply a sum in brackets by $x$ we, multiply each term in the sum by $x$. Most other operations, such as square rooting or finding the sine, you do NOT do separately to each term.
- Multiplying together two sets of brackets:

  If we are multiplying a sum by another sum, we need to multiply each term in the first sum by each term in the second sum.
- The difference between expressions and equations:

  With an equation, we can 'do the same thing' to both sides and maintain the truth of the equation. With an expression we cannot do anything which will change the expression's value. We can check whether we have succeeded by substituting values for any unknown elements. The expression should have the same value after simplification as it had beforehand.

**Worked example K.1**

Expand and simplify $(2+x)(1+3x+y)$.

| | |
|---|---|
| Multiply everything in the second bracket by 2 | $2+6x+2y$ |
| Multiply everything in the second bracket by $+x$ | $+x+3x^2+xy$ |
| Add the results together, collecting terms which are alike ($6x$ and $x$) | $=2+7x+2y+3x^2+xy$ |

## Exercise K

**1.** Write in as few terms as possible:

(a) $1+x+1+y$ (b) $2+2a-3a+a$

(c) $5x+xy-y^2+x^2-3xy$ (d) $(3z)^2+2z^2$

(e) $x(x+2)-2(x-1)$ (f) $x(x+y)+2y(x+z)+3z(y+x)$

2. Multiply out and simplify:

(a) $(x+3)(x+5)$ (b) $(z+1)(z-1)$ (c) $(2a+1)(3a-2)$

(d) $(x+1)(y+1)$ (e) $(x-3)^2$ (f) $(2x+y)^2$

(g) $(a-1)(a^2+a+1)$ (h) $(x+1)(x+2)(x+3)$ (i) $(x+y-1)^2$

3. Check the following simplifications by putting $x=2$ and $y=3$ into both forms:

(a) $(x+y)^2$ is the same as $x^2+y^2$

(b) $(3x)^2$ is the same as $9x^2$

(c) $\sqrt{xy}$ is the same as $\sqrt{x}\sqrt{y}$

(d) $\sqrt{y-x}$ is the same as $\sqrt{y}-\sqrt{x}$

(e) $\dfrac{1}{\left(\dfrac{x}{y}\right)}$ is the same as $\dfrac{y}{x}$

(f) $\dfrac{1}{x+y}$ is the same as $\dfrac{1}{x}+\dfrac{1}{y}$

(g) $\dfrac{x+y}{1+y}$ is the same as $\dfrac{x}{1}$

(h) $(x^y)^2$ is the same as $x^{2y}$

(i) $2^{x+y}$ is the same as $2^x+2^y$

(j) $(5x)^2$ is the same as $5x^2$

**EXAM HINT**

Just because a simplification works for some values you try does not guarantee that it is true. However, if you can find any value for which it fails to work then you can be sure the simplification is wrong

# Section L Solving equations and inequalities

The most important thing to remember when solving equations is that you have to do the same thing to both sides. A good tactic is to get all the unknowns on one side and everything else on the other side and then divide. Do not expect all the answers to be whole numbers .

### Worked example L.1

Solve $3+\frac{x}{5}=2-x$.

Getting rid of fractions is a good idea. Remember to multiply the whole of each side, not just the terms you are most interested in!

Multiply by 5:
$15+x=10-5x$

Get all terms containing $x$ on one side and everything else on the other
Try to get a positive coefficient of $x$

Add $5x$:
$15+6x=10$

Subtract 15:
$6x=-5$

$x=-\frac{5}{6}$

Linear inequalities can be solved in a similar way, with one important exception: when multiplying or dividing by a *negative* number, we must reverse the inequality sign.

### Worked example L.2

Solve the inequality $1-4x\le\frac{x}{2}+1$.

Getting rid of fractions is a good idea

Multiply by 2:
$2-8x\le x+2$

Get all terms containing $x$ on one side and everything else on the other

Subtract $x$:
$2-9x\le 2$

Subtract 2:
$-9x\le 0$

Divide by –9 and remember to change the sign

$x\ge 0$

The issue of changing the sign can be avoided if we always try to get a *positive* coefficient for $x$:

**Worked example L.3**

Solve the inequality $1-4x\le\frac{x}{2}+1$.

Get rid of fractions

$2-8x\le x+2$

Get all terms containing $x$ on one side and everything else on the other

Try to get a positive coefficient of $x$

Add $8x$:

$2\le 9x+2$

Subtract 2:

$0\le 9x$

Divide

$0\le x$

Conventionally $x$ is written first, so we need to rewrite this

$x\ge 0$

## Exercise L

**1.** Solve for $x$:

(a) $10+4x=7$ (b) $7+5x=9-x$ (c) $1-(3-5x)=8+2x$

(d) $\frac{4+5x}{3}+1=x$ (e) $3-2x=\frac{5+x}{3}$ (f) $1+\frac{1}{x}=3-\frac{1}{x}$

**2.** Solve these inequalities:

(a) $11+4x>29$ (b) $2+5x\le 4-x$ (c) $2x+1<4x-3$

(d) $3-\frac{x}{2}>2x+1$

# Section M Working with formulae

A formula is a set of instructions for finding one variable if you know some others. If we treat a formula as an equation we can change the subject of the formula from one variable to another. There are a few useful ideas we need to bear in mind when we are trying to do this.

- If the new subject occurs in any squares or square roots, isolate these before undoing them.
- Get all the terms involving the new subject on one side and everything else on the other side, then factorise.

**Worked example M.1**

Make $x$ the subject of the formula $y = \dfrac{3+x}{5-x}$.

Get rid of fractions and expand brackets

$$y(5-x) = 3+x$$
$$\Rightarrow 5y - xy = 3+x$$

Get everything involving $x$ on one side, everything else on the other side, then factorise

$$\Rightarrow 5y - 3 = xy + x$$
$$\Rightarrow 5y - 3 = x(y+1)$$
$$\Rightarrow \frac{5y-3}{y+1} = x$$

Another important tool is substituting one formula into another.

**Worked example M.2**

If $A = \pi r^2 + 2\pi rm$ and $r = 3m$, find $A$ in terms of $m$.

Wherever $r$ appears in the formula for $A$, replace with (3m), then expand all brackets

$$A = \pi(3m)^2 + 2\pi(3m)m$$
$$= 9\pi m^2 + 6\pi m^2$$
$$= 15\pi m^2$$

## Exercise M

1. (a) If $A = xy + y$, find $A$ when $x = 6$, and $y = \frac{1}{2}$.

   (b) If $B = (3p)^2$, find $B$ when $p = 2$.

   (c) If $C = \sqrt{x}(1 + y^2)$, find $C$ when $x = 9$ and $y = 2$.

2. Make $x$ the subject of each of these formulae:

   (a) $A = 3x + 7$ (b) $B = 3 - x^2$ (c) $C = 1 + \frac{1}{x}$

   (d) $D = 7 + \sqrt{1 - 3x}$ (e) $E = \frac{3 + x}{x - 2}$ (f) $F = \frac{10 + x}{3 + 2x}$

   (g) $G = \frac{1 + x^2}{2 + x^2}$ (h) $H = \frac{a + x}{8 + x}$ (i) $I = \frac{a + bx}{c + dx}$

3. Write $z$ in terms of $x$, giving your answer in a form without brackets:

   (a) $z = 5y + 5, y = 3x$ (b) $z = 6y - 1, y = 2x + 1$ (c) $z = 2y^2, y = 3x$

   (d) $z = y^2 + 2y + 5, y = x + 1$ (e) $z = 1 + \frac{y}{2}, y = 6x + 4$ (f) $z = \frac{1 + y}{1 - y}, y = x - 1$

# Section N Factorisation

Factorisation means taking a sum and re-expressing it as a product. This is often done by finding a factor common to each term in the sum.

**Worked example N.1**

Factorise $12ab^2 + 9a^2b$.

The largest factor in both terms is $3ab$

$= 3ab(4b + 3a)$

We must also be able to factorise quadratic expressions like $x^2 + bx + c$. If the coefficient of $x^2$ is 1 we look for a factorisation looking like $(x - p)(x - q)$.

We try to find $p$ and $q$ such that their product is $c$ and they add up to $b$. If there is a common term throughout the whole expression, do not forget to take that out first.

**Worked example N.2**

Factorise $x^2 - 7x + 12$.

–3, –4 are two numbers which add to give –7 and multiply to give 12

$= (x - 3)(x - 4)$

A special case of quadratic factorisation is the difference of two squares: $a^2 - b^2 = (a - b)(a + b)$. It is very useful to be able to recognise this.

**Worked example N.3**

Factorise $25x^2 - 9y^2$.

This is the difference of two squares, $a^2 - b^2$, with $a = 5x$, $b = 3y$

$= (5x)^2 - (3y)^2$
$= (5x - 3y)(5x + 3y)$

Sometimes it is possible to factorise an expression by first splitting it up into two parts and factorising each one separately. This sometimes reveals a common factor in both parts.

**Worked example N.4**

Factorise $ax + bx + ay + by$.

$= x(a + b) + y(a + b)$

A factor of (a + b) can be taken out

$= (a + b)(x + y)$

## Exercise N

**1.** Factorise the following:

(a) $8x+12y$ (b) $5y-10y^2$ (c) $18x^3y-4x$

(d) $14a^2b^2-7ab$ (e) $32x^3y+4x^2y^2+2xy^4$ (f) $15a^2b^2-12ab+9a^2$

**2.** Factorise the following:

(a) $x^2+2x-3$ (b) $x^2-2x-35$ (c) $a^2-8a-20$

(d) $b^2-81$ (e) $2x^2+10x+12$ (f) $5x^2-45$

**3.** Factorise the following:

(a) $xy+y+x+1$ (b) $x^2+xy+x+y$ (c) $ab+3a+2b+6$

(d) $6ab-8a+3b-4$ (e) $pq-p-q+1$ (f) $6p^2q-4pq-15p+10$

# Section O Algebraic fractions

- To simplify algebraic fractions:

  We can only multiply or divide every term on the top (numerator) and bottom (denominator) by the same thing. It often helps to factorise first. We cannot just cancel individual terms or elements in the numerator and denominator.
- To add algebraic fractions:

  We must find a common denominator. We can always do this by multiplying together the denominators of the fractions we are adding, although there are sometimes simpler common denominators.

  We then have to multiply the top and bottom of both fractions by an expression to get two fractions with a common denominator. When we add, we add together only the numerator and leave the denominator alone.
- Multiplying an algebraic fraction by a number:

  We multiply only the numerator by this number.
- Dealing with three-level algebraic fractions:

  The best thing to do here is to turn it into a four-level algebraic fraction, flip the bottom one and then multiply.

**Worked example 0.1**

Simplify $\frac{x^2+3x}{5x+15}$.

Factorise first

$$=\frac{x(x+3)}{5(x+3)}$$

Divide top and bottom by the common factor $(x + 3)$

$$=\frac{x}{5}$$

**Worked example 0.2**

Write as one fraction: $\frac{1}{x+1}-\frac{2x}{2x-1}$.

A common denominator can be found by multiplying together the two denominators

$$\frac{1}{x+1}-\frac{2x}{2x-1}$$

$$=\frac{1}{(x+1)}\frac{(2x-1)}{(2x-1)}-\frac{2x}{(2x-1)}\frac{(x+1)}{(x+1)}$$

$$=\frac{2x-1}{(x+1)(2x-1)}-\frac{2x(x+1)}{(2x-1)(x+1)}$$

There is no need to multiply out the factorised denominator

$$=\frac{2x-1-2x^2-2x}{(x+1)(2x-1)}$$

$$=\frac{-1-2x^2}{(x+1)(2x-1)}$$

## Worked example 0.3

Simplify $\dfrac{x+1}{\left(\dfrac{x+5}{4x}\right)}$.

Turn three-level fractions into four-level fractions

$$\frac{x+1}{\left(\dfrac{x+5}{4x}\right)}=\frac{\left(\dfrac{x+1}{1}\right)}{\left(\dfrac{x+5}{4x}\right)}$$

Division by a fraction is the same as multiplication by its reciprocal

$$=\left(\frac{x+1}{1}\right)\times\left(\frac{4x}{x+5}\right)$$

$$=\frac{4x^2+4x}{x+5}$$

## Exercise 0

**1.** Simplify where possible:

(a) $\dfrac{5}{5x}$ (b) $\dfrac{3x+6}{3}$ (c) $\dfrac{x^2+5x}{5+x}$

(d) $\dfrac{x^2+1}{x+1}$ (e) $\dfrac{x^2+2x+1}{x^2+3x+2}$ (f) $\dfrac{a^2-9}{a^2+6a+9}$

**2.** Write as a single fraction:

(a) $\dfrac{x}{5}+\dfrac{3x+1}{6}$ (b) $\dfrac{x}{2}+\dfrac{2}{x}$ (c) $\dfrac{1}{x+1}+\dfrac{1}{x+2}$

(d) $\dfrac{3}{x}-\dfrac{1-x}{1+x}$ (e) $\dfrac{3x-1}{x+2}+\dfrac{x+1}{2x-1}$ (f) $\dfrac{x+4}{3x-1}-\dfrac{x-2}{3x+1}$

**3.** Simplify:

(a) $\dfrac{3}{\left(\dfrac{5}{x}\right)}$ (b) $\dfrac{x+1}{\left(\dfrac{x+1}{4}\right)}$ (c) $\dfrac{\left(\dfrac{1}{x+5}\right)}{x-2}$

(d) $\dfrac{\left(\dfrac{2x+7}{x-2}\right)}{x-1}$ (e) $\dfrac{\left(\dfrac{3x}{x-4}\right)}{2x}$ (f) $\dfrac{3x+3}{\left(\dfrac{x+1}{2}\right)}$

# Section P Cartesian geometry

We can describe the position of any point in a plane, relative to an origin, using two numbers, which we call coordinates. The $x$-coordinate describes how far to the right the point is and the $y$-coordinate describes how far up a point goes. A negative coordinate means the point is left or down.

We can calculate the gradient between two points, which gives a measure of the steepness of the line connecting the points. If the gradient is positive, the $y$-coordinate increases from left to right. If it is negative, then the $y$-coordinate decreases from left to right. If the two points have coordinates $(x_1, y_1)$, and $(x_2, y_2)$, then:

KEY POINT P.1

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$

There are also formulae to calculate the coordinates of the point exactly half way (the midpoint) between two coordinates, and to calculate the distance between two points. These are given in the Formula booklet.

KEY POINT P.2

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$\text{Distance between two points} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

**Worked example P.1**

The point $A$ has coordinates $(1, 5)$. The point $B$ has coordinates $(-3, 3)$. What is the gradient of the line connecting $A$ and $B$, and what are the coordinates of the midpoint of $A$ and $B$? What is the distance between $A$ and $B$?

Use the formula for gradient

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 5}{-3 - 1} = \frac{-2}{-4} = \frac{1}{2}$$

Use the formula for midpoint

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{1 + -3}{2}, \frac{5 + 3}{2}\right) = \left(\frac{-2}{2}, \frac{8}{2}\right) = (-1, 4)$$

continued . . .

Use the formula for distance

$$\begin{aligned} Distance &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-3-1)^2 + (3-5)^2} \\ &= \sqrt{(-4)^2 + (-2)^2} \\ &= \sqrt{16+4} \\ &= \sqrt{20} \end{aligned}$$

## Exercise P

**1.** Find the gradient of the line connecting:

(a) $(1, 4)$ and $(3, 7)$  (b) $(4, -2)$ and $(9, 2)$  (c) $(-3, -3)$ and $(5, -3)$

**2.** Find the midpoint of the following points:

(a) $(1, 4)$ and $(3, 7)$  (b) $(4, -2)$ and $(9, 2)$  (c) $(-3, -3)$ and $(5, -3)$

**3.** Find the distance between the two points:

(a) $(1, 4)$ and $(3, 7)$  (b) $(4, -2)$ and $(9, 2)$  (c) $(-3, -3)$ and $(5, -3)$

# Section Q Simultaneous equations

If we have two equations and two unknowns, then we can often solve them to find just one set of values that works in both equations. There are two algebraic methods for doing this:

## 1. Elimination

We can add or subtract entire equations (and multiples of them) to entirely eliminate one of the variables.

### Worked example Q.1

Solve the simultaneous equations:

$$\begin{cases} 3x+5y=0 \\ 6x-2y=10 \end{cases}$$

$3x+5y=0$ (1)
$6x-2y=10$ (2)

Look for a way to multiply one equation to match the coefficient of a variable in another

Labelling the equations with numbers in brackets makes it clear to the reader (and to you) how you are manipulating the equations

(1) × 2 $\quad 6x+10y=0$ (3)

(3) − (2) $\quad 12y=10$

$\Rightarrow \quad y=\dfrac{5}{6}$

into (1) $\quad 3x+\dfrac{25}{6}=0$

$\Rightarrow \quad x=-\dfrac{25}{18}$

## 2. Substitution

We can rearrange one of the equations so that one of the variables is the subject and then substitute this into the other equation. Again, we will have formed an equation with just one variable. The advantage of this method is that it can be used with some non-linear simultaneous equations.

### Worked example Q.2

Solve the simultaneous equations:

$x+y=7$

$x^2+y^2=169$

Rearrange the simpler equation to make one variable the subject

$y=7-x$

continued . . .

Substitute into the second equation

$$x^2+(7-x)^2=169$$
$$\Leftrightarrow x^2+49-14x+x^2=169$$
$$\Leftrightarrow 2x^2-14x=120$$
$$\Leftrightarrow x^2-7x=60$$
$$\Leftrightarrow x^2-7x-60=0$$
$$\Leftrightarrow (x-12)(x+5)=0$$
$$\Leftrightarrow x=12 \text{ or } x=-5$$

Use $y=7-x$ to find $y$ for each solution for $x$

When $x=12,\ y=-5$

When $x=-5,\ y=12$

Once we have solved the simultaneous equations, we can interpret the solution as the coordinates of the point (or points) where the two lines intersect.

## Exercise Q

**1.** Find the intersection point of the lines given. Use whichever method you prefer.

(a) $y=3x+1$ and $y=5x-3$

(b) $x+3y=1$ and $y+x=2$

(c) $2x+2y=1$ and $x-y=2$

(d) $3y+2x=5$ and $x-y=0$

(e) $2x-y-1=0$ and $x+y=5$

(f) $y=2x-1$ and $y+2x=7$

**2.** Solve the following simultaneous equations:

(a) $x+y=7$ and $x^2+y^2=25$

(b) $y-x=10$ and $x^2+2y=44$

(c) $y^2=2x+1$ and $2x+3y=17$

(d) $x+2y=13$ and $xy=20$

## Section R Straight line graphs

If we are given a rule connecting the $x$-coordinate and the $y$-coordinate then, only some points on the plane will satisfy them. If the rule is of the form $y = mx + c$, then those points will lie in a straight line with gradient $m$ and the line will meet the $y$-axis at $(0, c)$. If we want to find information about a straight line, it is best if we first rearrange it into the form $y = mx + c$.

To define a straight line we need either two points that it passes through or one point and a gradient.

KEY POINT R.1

A line passing through the point $(x_1, y_1)$ with gradient $m$ has the equation:

$$y - y_1 = m(x - x_1)$$

If you are given two points, we can work out the gradient between them and then use the above equation.

If there are two lines, one with gradient $m_1$ and the other with gradient $m_2$, we can use this information to see how the two lines are arranged relative to each other.

KEY POINT R.2

If $m_1 = m_2$ then the two lines are parallel

If $m_1 m_2 = -1$ then the two lines are perpendicular

**Worked example R.1**

A line has equation $3x + 2y = 2$.

Find the equation of a line perpendicular to this line through the point $(1,2)$.

Rearrange into the form $y = mx + c$ to find the gradient

$$2y = 2 - 3x$$

$$\Leftrightarrow \quad y = 1 - \frac{3}{2}x$$

So the gradient is $\frac{-3}{2}$

The product of perpendicular gradients is $-1$

The gradient of the perpendicular line satisfies

$$-\frac{3}{2}m = -1$$

$$\Rightarrow \quad m = \frac{2}{3}$$

We now have a point and a gradient

$$y - y_1 = m(x - x_1)$$

$$\therefore \quad y - 2 = \frac{2}{3}(x - 1)$$

$$\Leftrightarrow \quad y = \frac{2}{3}x + \frac{4}{3}$$

## Exercise R

**1.** Find the gradient and $y$-axis intersection of the following straight lines:

(a) $y = 2x - 7$ (b) $y = 5 - x$ (c) $3x + y = 9$

(d) $y + 5 = 3(x - 1)$ (e) $5y = 10x + 1$ (f) $3y + 2x = 0$

**2.** Find the equation of each line in the form $ax + by = c$ where $a$, $b$ and $c$ are integers:

(a) through $(1,3)$ with gradient $7$

(b) through $(2,2)$ with gradient $-\frac{1}{2}$

(c) through $(0,0)$ with gradient $-5$

(d) through $(0,6)$ and $(6,0)$

(e) through $(1,6)$ and $(9,6)$

(f) through $(4,9)$ and $(4,18)$

**3.** Find in the form $y = mx + c$ the equation of:

(a) a line parallel to $y = 3x - 1$ through $(1,1)$

(b) a line parallel to $x + y = 5$ through $(2,4)$

(c) a line parallel to $3x - 2y + 4 = 0$ through $(0,0)$

(d) a line perpendicular to $y = x + 5$ through $(5,1)$

(e) a line perpendicular to $x + 2y = 1$ through $(6,0)$

(f) a line perpendicular to $3x - 5y - 2 = 0$ through $(3,1)$

# Section S Geometric transformations

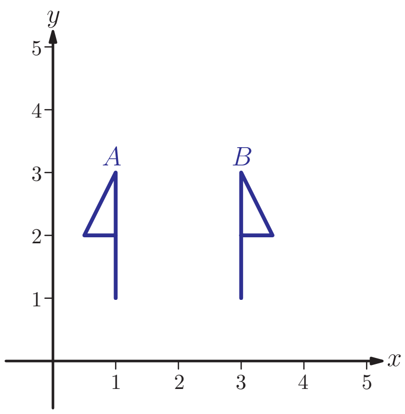
There are four transformations which you need to know: rotations, reflections, translations and enlargements.

It is important to know what information is required to *fully* define each of these transformations.

| Transformation | Information required |
|---|---|
| Rotation | Angle<br>Direction (clockwise or anticlockwise)<br>Coordinates of the centre of rotation |
| Reflection | Equation of the line of reflection |
| Translation | Vector of the translation |
| Enlargement | Centre of enlargement<br>Enlargement scale factor |

**Worked example S.1**

Describe fully the single transformation required to change shape $A$ to shape $B$.




It is a reflection in the line $x = 2$

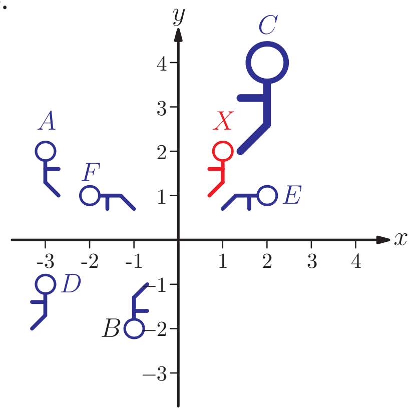
## Exercise S

**1.** Describe the single transformation that takes shape $X$ to:

(a) Shape $A$ (b) Shape $B$

(c) Shape $C$ (d) Shape $D$

(e) Shape $E$ (f) Shape $F$



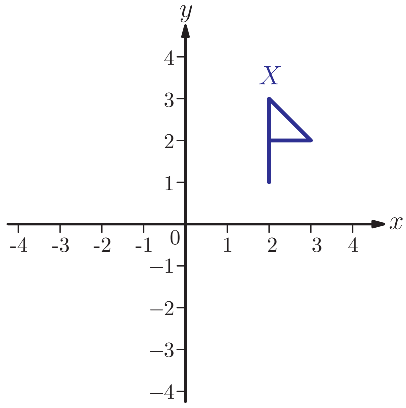
**2.** Draw and label the image of the shape $X$ under each transformation:

(a) Reflection in the line $y = 1$.

(b) Rotation $90^\circ$ anticlockwise centred on $(-1,0)$.

(c) Translation $\begin{pmatrix} -4 \\ 0 \end{pmatrix}$.

(d) Enlargement factor 0.5 centre $(0,1)$.



# Section T Similar and congruent shapes

Two shapes are similar if one is an enlargement of the other (even if it is in a different position and orientation). There is a constant scale factor of enlargement from one to the other so the ratio of a side on one of the shapes to the corresponding side on the other shape is always the same.

Two shapes are congruent if they are similar *and* the same size, even if they are in different positions and orientations. Remember that if one shape is a reflection of another then they are congruent.

Triangles are particularly important in these situations. If two triangles, $P$ and $Q$, are similar then each angle in $P$ is the same as each angle in $Q$. We describe this condition as AAA, meaning that there are three pairs of equal angles. With congruent triangles we are also interested in pairs of equal length sides, which are denoted by the letter S. The conditions for two triangles to be congruent are:

SSS – each side in one triangle matches a side in the other

ASA – two angles and the enclosed side in one triangle match two angles and the enclosed side in the other

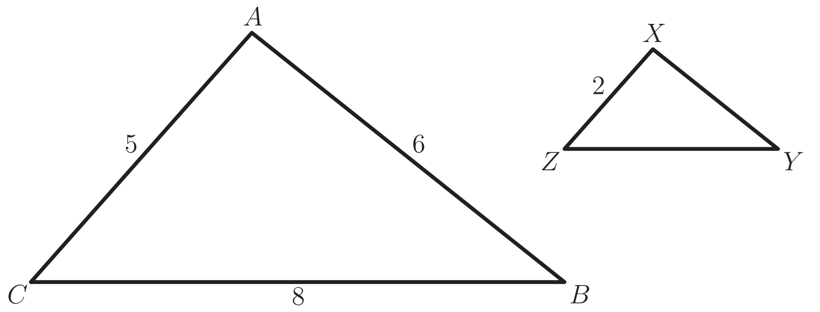
SAS – two sides and the enclosed angle in one triangle match two sides and the enclosed angle in the other

The order of letters here is important – ASS is not a congruence condition, as there is sometimes more than one triangle that can be drawn from this information. There is a common exception to this – if the angle is a right angle then we can use a fourth congruence condition:

RHS – in right-angled triangles, the hypotenuse and one other side in one triangle match the hypotenuse and one other side in the other triangle

**Worked example T.1**

Triangles $ABC$ and $XYZ$ are similar. Find the length of the side $XY$.




First find the scale factor of enlargement by comparing equivalent sides

$$\text{Scale factor} = \frac{XZ}{AC} = \frac{2}{5}$$

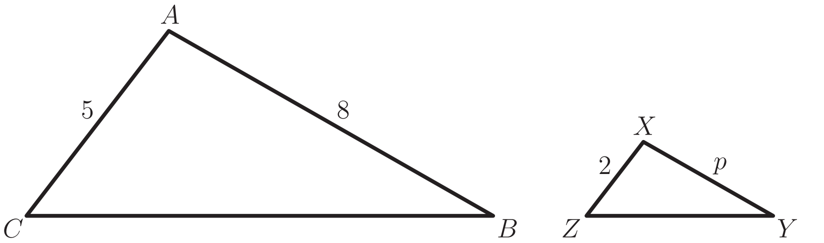
Apply the scale factor to find $XY$

$$\text{So } XY = \frac{2}{5} \times AB = \frac{2}{5} \times 6 = 2.4$$

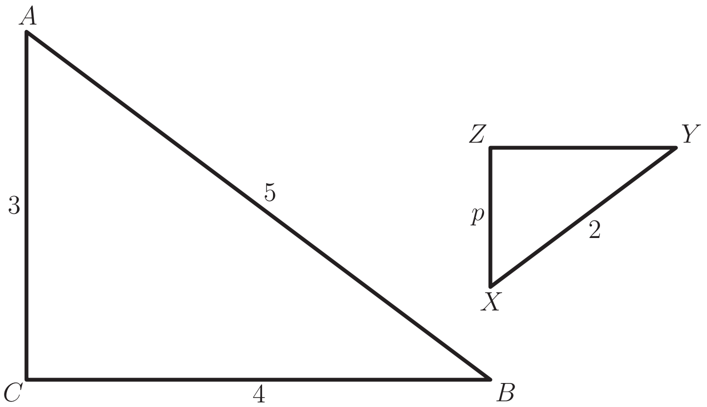
## Exercise T

**1.** In the following diagrams triangle $ABC$ is similar to triangle $XYZ$, with angle $\hat{A} = \hat{X}$ and $\hat{B} = \hat{Y}$. Find the length marked $p$.

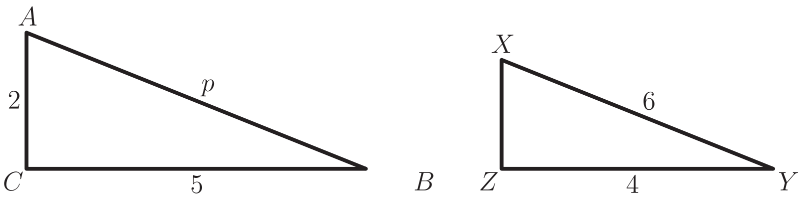
(a)




(b)

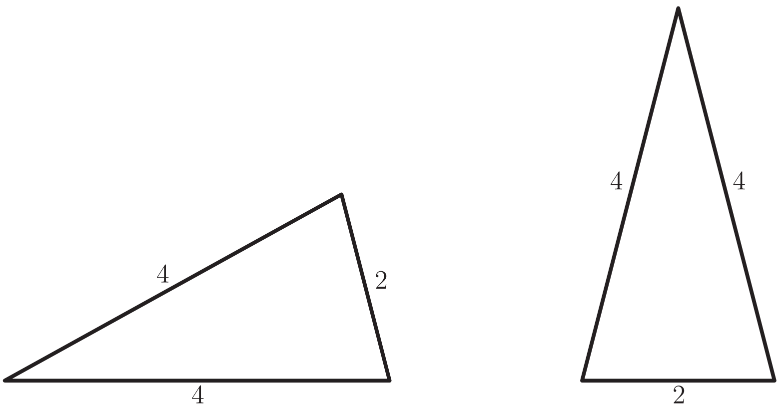



(c)

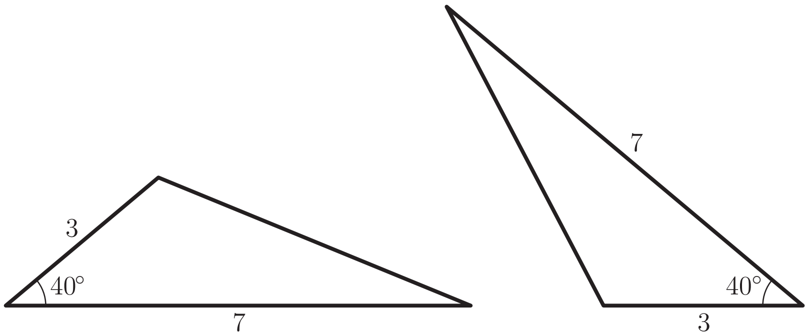



**2.** State whether information given is sufficient to show that the pairs of triangles below are congruent. If they are, state the congruence condition.

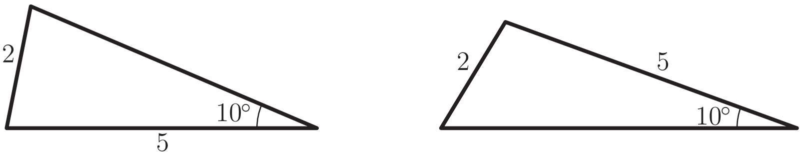
(a)



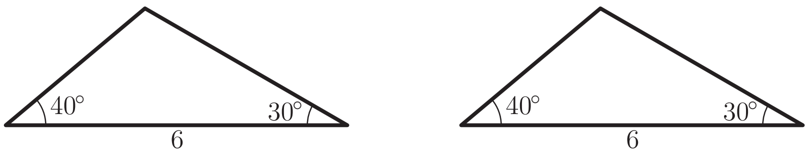
(b)




(c)




(d)



# Section U Basic trigonometry

There is a convention for labelling triangles. The angles are given capital letters and the opposite sides are labelled with the same letter but in lower case.

In a right-angled triangle, the longest side is always opposite the right angle and it is called the hypotenuse. You must be familiar with the following rules, all of which ONLY apply to right-angled triangles.

KEY POINT U.1

Pythagoras' theorem:

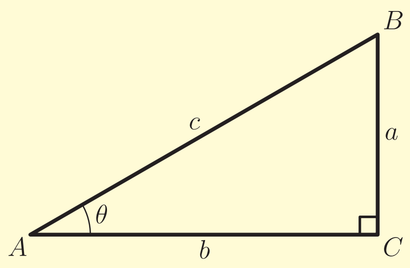
$$c^2 = a^2 + b^2$$

Trigonometric ratios:

$$\sin\theta = \frac{a}{c} = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{b}{c} = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{a}{b} = \frac{\text{opposite side}}{\text{adjacent side}}$$

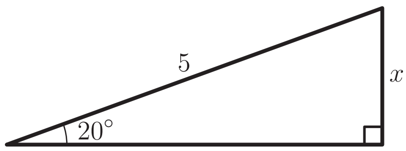



You should also know that Pythagoras' theorem works in reverse; if you have a triangle with $c^2 = a^2 + b^2$, then it must be right-angled with the right angle at $C$.

If we know the lengths in a right-angled triangle, we can then use these to find the angles. To do this we have to 'undo' one of the trigonometric ratios above to get just $\theta$. This is done using the $\sin^{-1}$, $\cos^{-1}$ or $\tan^{-1}$ buttons on your calculator.

**Worked example U.1**

Find the length $x$ in the diagram below:




First decide which trig ratio to use. The relevant sides are the side opposite the angle and the hypotenuse, so this means sin

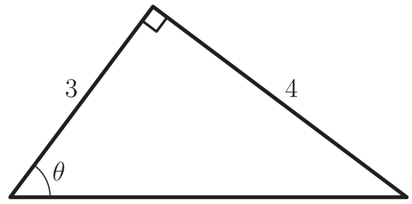
$$\sin 20° = \frac{x}{5}$$

$$x = 5\sin 20°$$

$$= 1.71 \text{ (3SF)}$$

## Worked example U.2

Find the angle $\theta$ in the diagram:



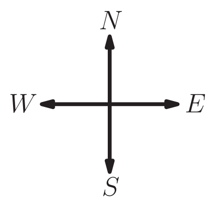

First decide which trig ratio to use. The relevant sides are the side opposite the angle and the side adjacent to the angle, so this means tan

$$\tan\theta = \frac{4}{3}$$

To find $\theta$ we need to undo the tangent operation

$$\theta = \tan^{-1}\left(\frac{4}{3}\right)$$
$$= 53.1^\circ \text{ (3SF)}$$

In trigonometry questions there may be references to compass points. You need to know these:

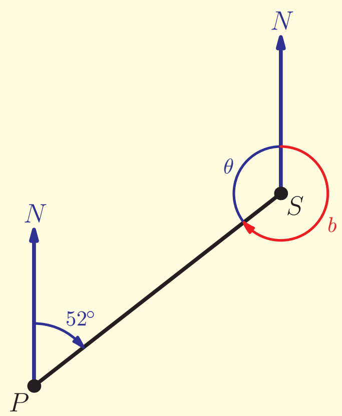



Questions may also refer to bearings. These are angles measured clockwise from North. They are given as three digit integers (for example 024° or 312°). (It is important to remember that any lines pointing North which you draw onto a diagram will be parallel.)

## Worked example U.3

The bearing of $S$ from $P$ is 052°. Find the bearing of $P$ from $S$.

Draw a diagram and label required angles



continued . . .

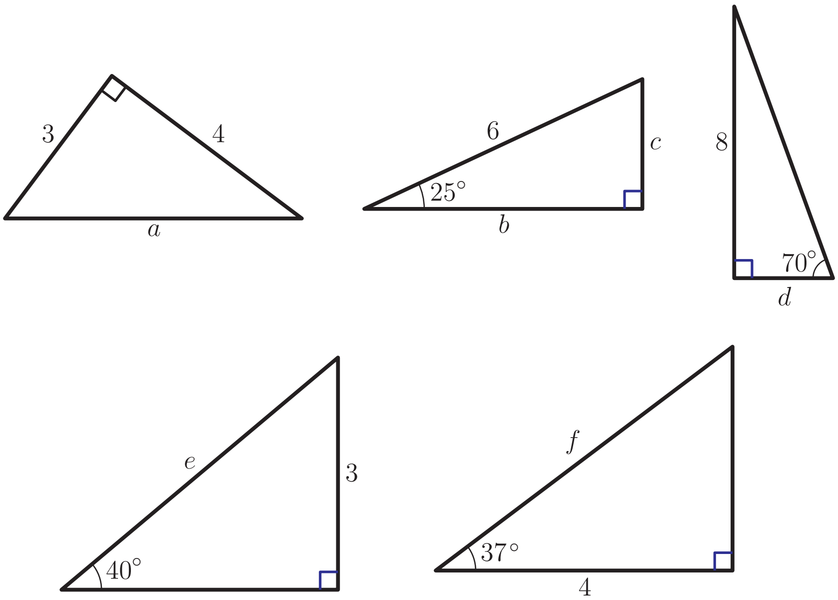
Angles 52° and $\theta$ add up to 180°
(See Prior learning Section W part 2)

$\theta = 180 - 52 = 128°$

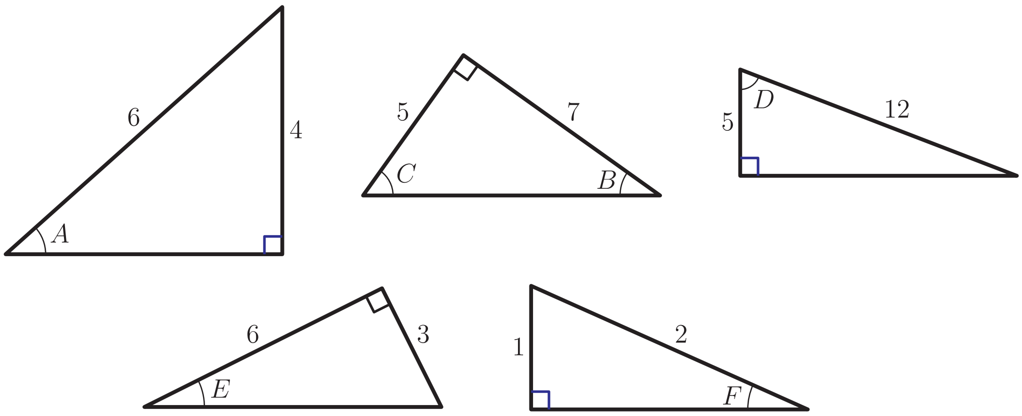
$\therefore b = 360 - \theta = 232°$

## Exercise U

**1.** Find the unknown lengths:




**2.** Find the unknown angles:




**3.** A ship is 12.5 km East and 10 km North of a lighthouse. Find the bearing of the ship from the lighthouse.

# Section V Area, perimeter and volume

You need to be familiar with the following plane shapes and their properties (see also Section W):

- triangles
- quadrilaterals (for example: square, rectangle, parallelogram, rhombus, trapezium, kite)

In the United States of America, the word 'trapezium' generally refers to an irregular quadrilateral, while 'trapezoid' is the term for a quadrilateral with (at least) one pair of parallel sides. Outside the USA, 'trapezium' refers to a quadrilateral with at least one pair of parallel sides, and this is the meaning we shall use.

and these three-dimensional objects:

- prisms (for example: cube, cuboid, triangle-based prism, cylinder)
- pyramids (for example: tetrahedron, square-based pyramid, cone)
- spheres.

The following formulae are given in the Formula booklet. Remember that you still have to be able to identify the shape in the question.

Area of a parallelogram $= (b \times h)$, where $b$ is the base, $h$ is the height.

Area of a triangle $A = \frac{1}{2}(b \times h)$, where $b$ is the base, $h$ is the height.

Area of a trapezium $= \frac{1}{2}(a+b)h$, where $a$ and $b$ are the parallel sides, $h$ is the height.

Area of a circle $= \pi r^2$, where $r$ is the radius.

Circumference of a circle $= 2\pi r$, where $r$ is the radius.

Volume of a pyramid $V = \frac{1}{3}$(area of base $\times$ vertical height).

Volume of a cuboid $= l \times w \times h$, where $l$ is the length, $w$ is the width, $h$ is the height.

Volume of a cylinder $V = \pi r^2 h$, where $r$ is the radius, $h$ is the height.

Area of the curved surface of a cylinder $= 2\pi rh$, where $r$ is the radius, $h$ is the height.

Volume of a sphere $= \frac{4}{3}\pi r^3$, where $r$ is the radius.

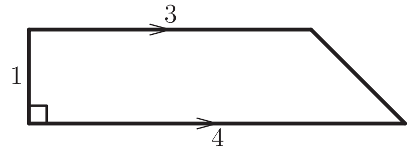
Volume of a cone $= \frac{1}{3}\pi r^2 h$, where $r$ is the radius, $h$ is the height.

A kite is the other shape which is mentioned in the syllabus but the area is **not** given in the Formula booklet.

Area of a kite $= \frac{1}{2}(d_1 \times d_2)$ where $d_1$ and $d_2$ are the lengths of the diagonals.

## Worked example V.1

Find the area of the following shape:




Identify the shape

Quadrilateral with two parallel sides: Trapezium

Find what is needed for the formula

$a = 3, b = 4, h = 1$

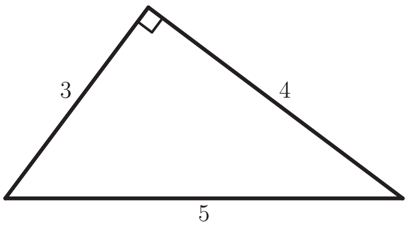
Quote formula, then substitute the values and calculate

$$\begin{aligned} A &= \frac{1}{2}(a+b)h \\ &= \frac{1}{2}(3+4)\times 1 \\ &= 3.5 \end{aligned}$$

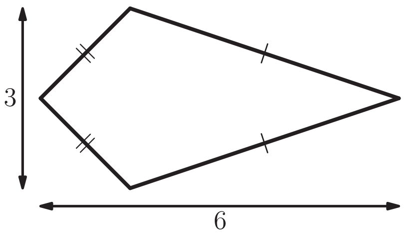
## Exercise V

**1.** Find the area of each of the following shapes:

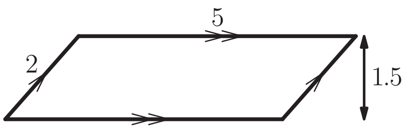
(a)



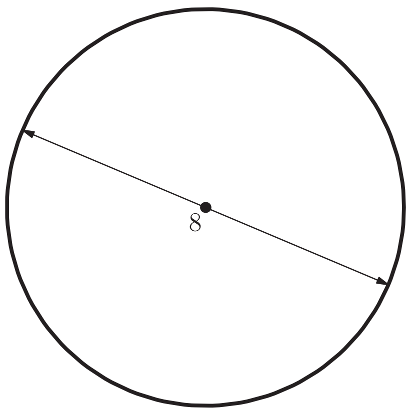

(b)




(c)

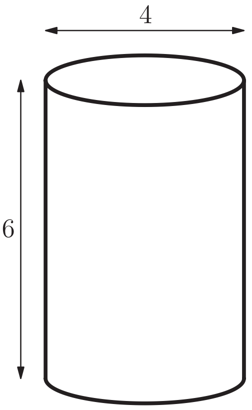



(d)

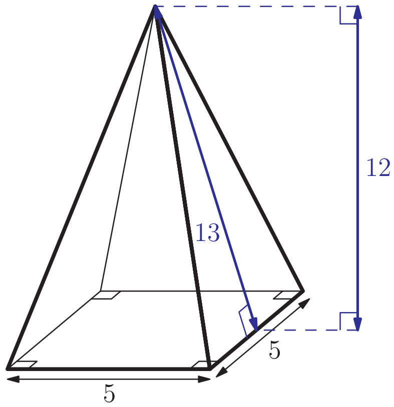


**2.** Find the volume of each of the following symmetrical shapes:

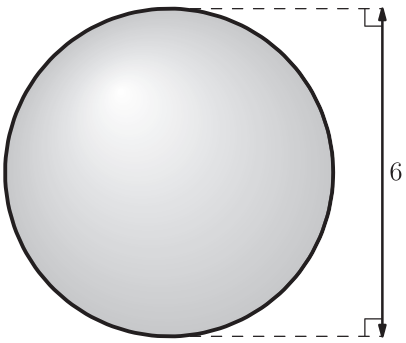
(a)



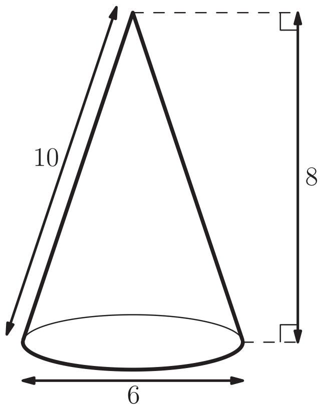

(b)




(c)




(d)

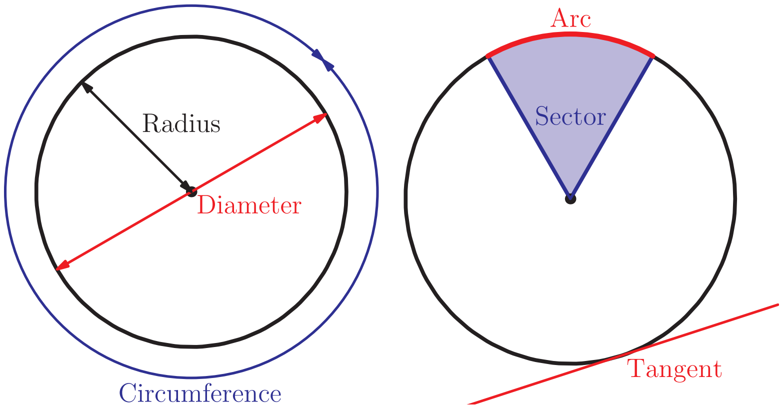


# Section W Useful geometrical facts

The material in this section is not mentioned explicitly anywhere in the syllabus, but these facts have been required in exam questions.

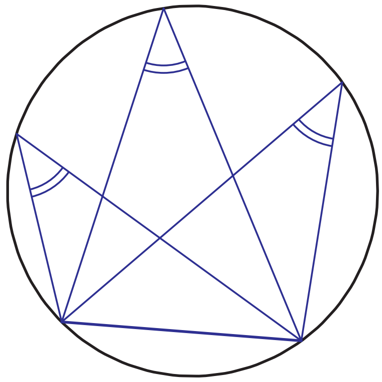
## Part 1: Circles

There are several pieces of terminology you need to know:

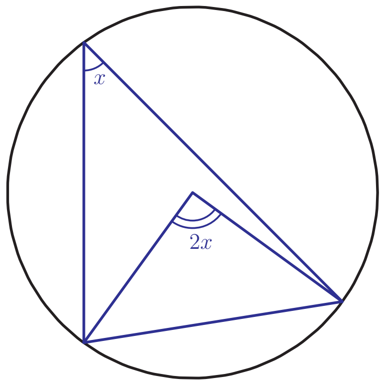



There are several theorems about angles in circles which you need to know:

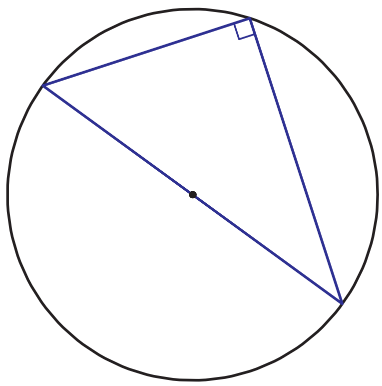
Angles in the same segment are equal:



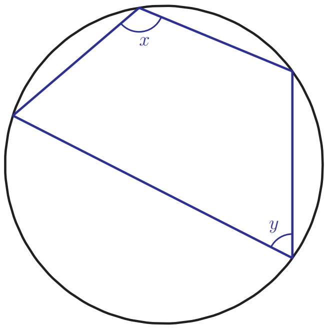
The angle at the centre is twice the angle at the circumference:




The angle in a semicircle is 90°:

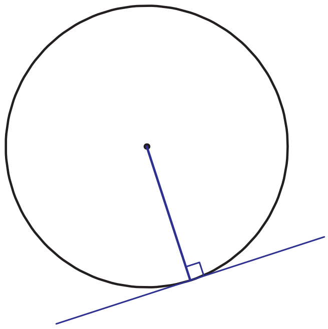


Opposite angles in a cyclic quadrilateral (a quadrilateral with all four corners on the circumference of a circle) add up to 180°:



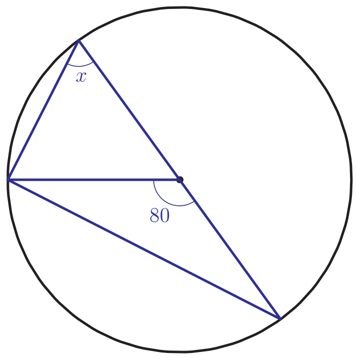

$x + y = 180°$

A tangent meets its radius at right angles:



**Worked example W.1**

Find the angle marked $x$ in this diagram.

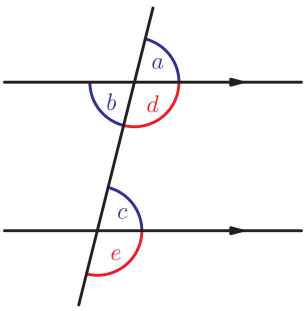



Angle at the centre is twice the angle at the circumference.
$80 = 2x$
$40 = x$

## Part 2: Quadrilaterals

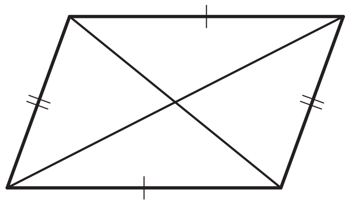
You need to know about angles on parallel lines.

- $a = b = c$
- $d = e = 180° - a$



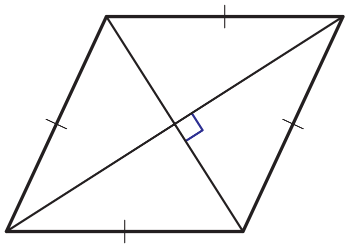
These facts about parallelograms are often needed:

- opposite sides are equal
- opposite angles are equal
- adjacent angles add up to 180°
- diagonals bisect each other.



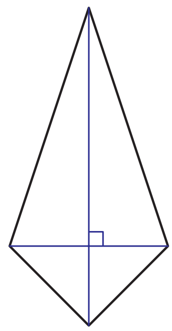
A rhombus is a type of parallelogram in which all four sides are equal. There are two further useful facts about a rhombus:

- the diagonals are perpendicular to each other
- the area is equal to $\frac{d_1 d_2}{2}$, where $d_1$ and $d_2$ are the lengths of the diagonals.



A kite is a quadrilateral in which pairs of adjacent sides are equal:

- the diagonals are perpendicular to each other
- one of the diagonals is the line of symmetry
- the area is equal to $\frac{d_1 d_2}{2}$, where $d_1$ and $d_2$ are the lengths of the diagonals.

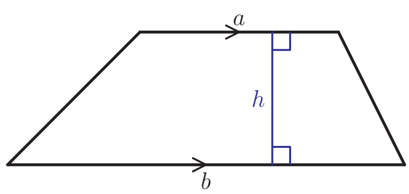


A trapezium is a quadrilateral which has one pair of parallel sides:

- two pairs of adjacent angles add up to 180°:

$$\hat{A}+\hat{D}=180° \text{ and } \hat{B}+\hat{C}=180°$$

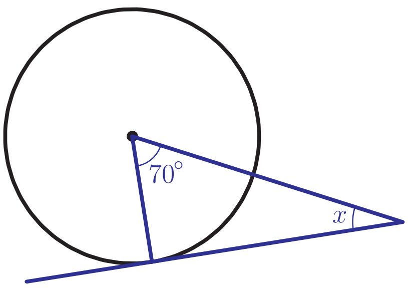
- the area is equal to $\frac{a+b}{2}h$.



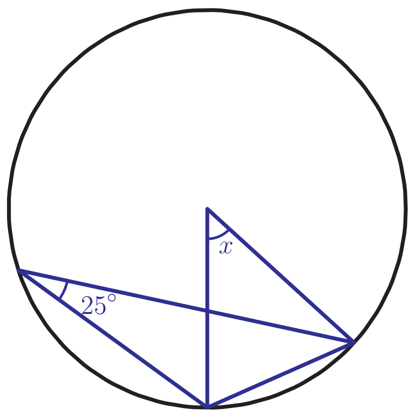

## Exercise W

**1.** Find the angle marked $x$ in the following diagrams, giving reasons for your answers.

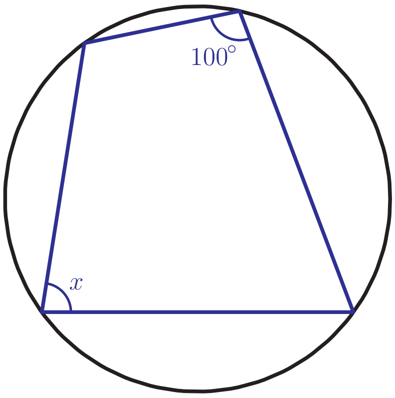
(a)



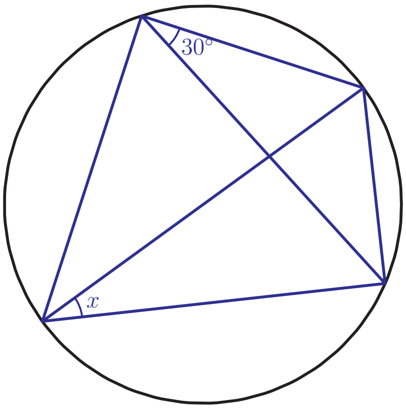

(b)



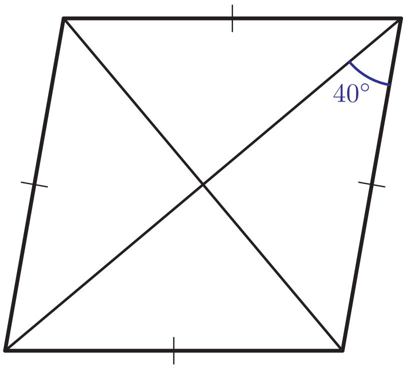
(c)




(d)




**2.** Find the angles of the rhombus:



# Section X Statistical diagrams

You should be able to interpret:

- Pie charts:

  These represent proportions by sizes of sectors.

- Bar charts:

  These represent frequencies by the heights of bars.

- Pictograms:

  Frequencies are proportional to the number of icons displayed.

- Stem and leaf diagrams:

  The last one (occasionally two) significant figure(s) of each data item is listed from a 'stem' of all previous significant figures.

- Line graphs:

  A line graph is normally used with time along the $x$-axis and the values recorded along the $y$-axis. Consecutive data points are joined by a straight line, representing an assumption that the data change at a constant rate in the time interval.

- Frequency histograms:

  These are like bar charts, but used for continuous data, where each bar covers a range of values, called the class (or group) interval. There are no gaps between bars. Heights of bars represent frequencies.

- Cumulative frequency graphs:

  Cumulative frequency is the total number of data items up to a certain value. A cumulative frequency graph has data values on the $x$-axis and cumulative frequency on the $y$-axis.

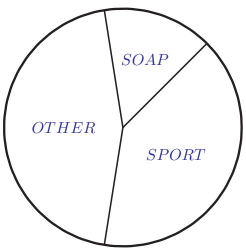
**Worked example X.1**

Below are a pie chart and a bar chart showing the types of television programmes watched by girls and boys in a class.

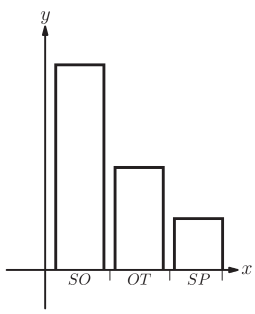
(a) In which group did a greater proportion of students watch soap operas?

(b) Did more boys than girls watch sport?

Boys:




Girls:




(a) Soap is the least popular category for boys but the most popular for girls so a greater proportion of girls watch soaps.

(b) We cannot say because the pie chart shows proportions not numbers.

## Worked example X.2

The table shows data for masses of eggs, measured in grams:

| Mass of eggs, in g | Frequency |
|---|---|
| [100,120[ | 26 |
| [120,140[ | 52 |
| [140,160[ | 84 |
| [160,180[ | 60 |
| [180,200[ | 12 |

(a) Convert this into a cumulative frequency table.

(b) Draw a cumulative frequency graph for the masses of eggs.

There are:
26 eggs up to 120 g,
26 + 52 = 78 eggs
up to 140 g, and so on

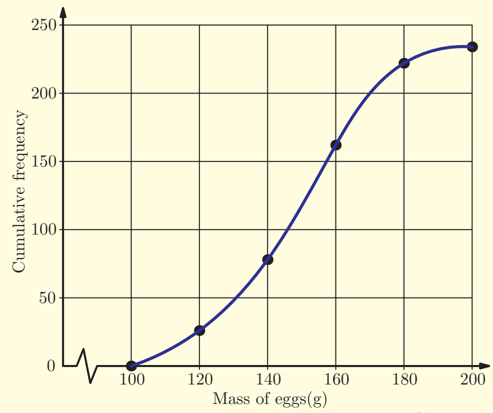
(a)

| Mass of eggs, in g | Cumulative frequency |
|---|---|
| 120 | 26 |
| 140 | 78 |
| 160 | 162 |
| 180 | 222 |
| 200 | 234 |

Cumulative frequency is plotted against the end point of the interval, so the first row of the table gives the point (120, 26)

There is always an additional data point to be plotted: the lowest lower bound and zero cumulative frequency

(b)




## Worked example X.3

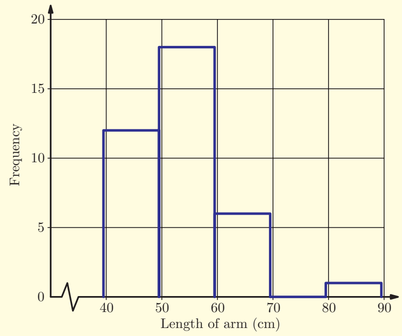
(a) Draw a frequency histogram to represent the following data:

| Length of arm (to the nearest cm) | Frequency |
|---|---|
| $39.5 \le l < 49.5$ | 12 |
| $49.5 \le l < 59.5$ | 18 |
| $59.5 \le l < 69.5$ | 6 |
| $69.5 \le l < 79.5$ | 0 |
| $79.5 \le l < 89.5$ | 1 |

continued . . .

(b) Estimate how many people have arms between 49.5 cm and 64.5 cm in length.

(a)

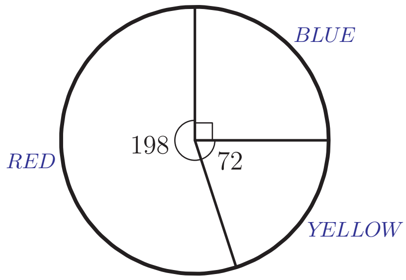



(b) $18+\frac{1}{2}(6)=21$ people

The range $49.5 \le l < 64.5$ covers the entire second group and half of the third group

## Exercise X

**1.** The following pie chart represents the favourite colours of 100 16-year-old pupils.




(a) How many pupils' favourite colour is:

(i) Yellow (ii) Blue (iii) Red?

(b) In a survey of the favourite colour of 12-year-olds, the results were:

| Colour | Number |
|---|---|
| Blue | 18 |
| Yellow | 12 |
| Red | 42 |

In a pie chart, what angle would the sector representing each of these colours measure:

(i) Blue (ii) Yellow (iii) Red

(c) Draw bar charts showing the results for 16-year-olds and 12-year-olds.

2. (a) Draw a cumulative frequency graph for the following data, where $x$ is the time taken to complete a puzzle in seconds.

| $x$ | Frequency |
|---|---|
| [0,15[ | 19 |
| [15,30[ | 15 |
| [30,45[ | 7 |
| [45,60[ | 5 |
| [60,90[ | 4 |

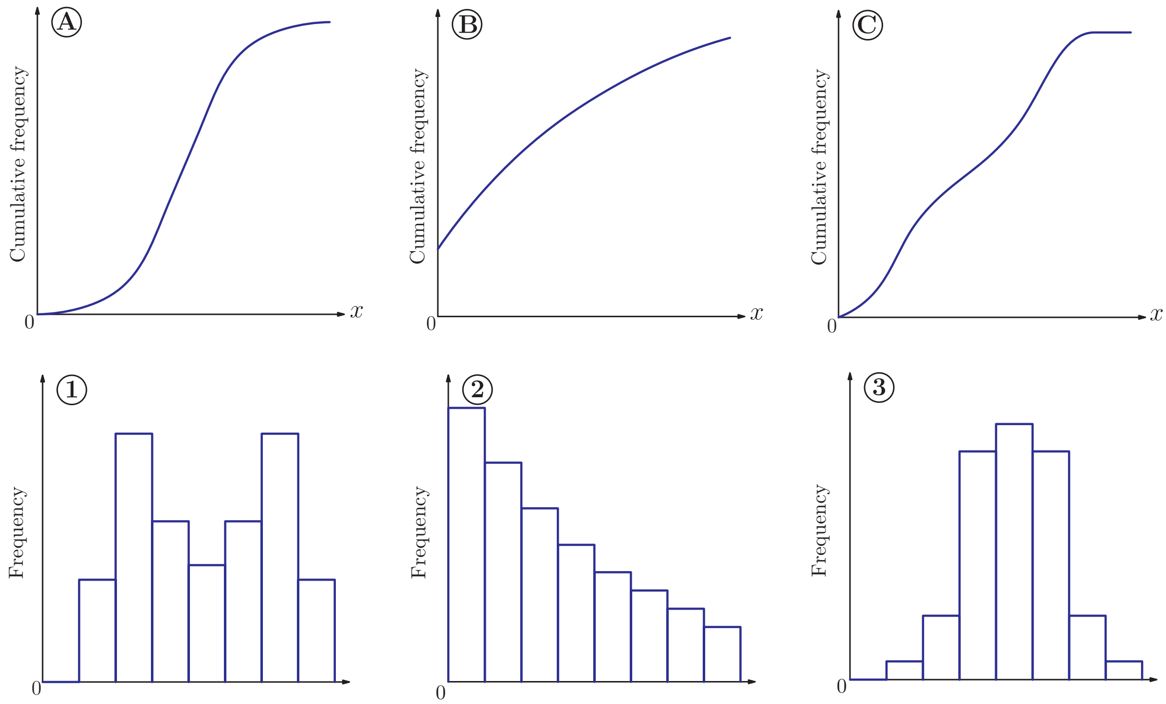
(b) Hence estimate how many people take less than 40 seconds to complete the puzzle.

3. (a) Draw a frequency histogram to represent the following data for the ages of children in a hospital ward.

| $x$ | Frequency |
|---|---|
| 0 to 2 | 12 |
| 3 to 5 | 15 |
| 6 to 8 | 7 |
| 9 to 11 | 6 |
| 12 to 14 | 3 |

(b) Estimate how many children under 8 there are in the ward.

4. Match each histogram with the cumulative frequency diagrams coming from the same data.



# Section Y Statistical calculations

Mean, median and mode are all measures of the centre (average) of a group of data.

- The mean is the total of all the data divided by the number of data items.
- The median is the value of the middle item when all the data is arranged in order. If there are two middle items, the mean of these two is taken.
- The mode is the value of the most common data item. There may be more than one mode.
- Range and inter-quartile range are measures of the spread of data.
- The range is the difference between the largest and the smallest data values.
- The inter-quartile range is the difference between the upper and lower quartiles. It contains the middle 50% of the data.

Quartiles and percentiles

- When all the data is arranged in ascending order, the data item one quarter of the way up is called the lower quartile (abbreviated to LQ or $Q_1$) and the one three quarters of the way up is called the upper quartile (UQ or $Q_3$). To find these quartiles we split the ordered data into two halves (discarding the central item if there is an odd number of data points) and then follow the same procedure as for finding the median. (In the abbreviation system mentioned above, the median can be called $Q_2$, the minimum value $Q_0$ and the maximum value $Q_4$).
- The $P$th percentile is the data item which has $P$% of the data below it. For example, $Q_1$ is the 25th percentile.
- Quartiles and percentiles can be found from a cumulative frequency graph.

Your calculator can find most of the these statistical measures, see Calculator Skills sheets 12–14 on the CD-ROM.

**Worked example Y.1**

Find the mean, median and mode of the data 7, 5, 6, 8, 5.

| | |
|---|---|
| Add up all the data | Mean:<br>$7+5+6+8+5=31$ |
| Count how many data items there are | 5 data items<br>Mean $=\frac{31}{5}=6.2$ |
| Arrange the data in order | Median:<br>5, 5, 6, 7, 8 |
| Decide which is the middle data item | With 5 data items the 3rd item is in the middle.<br>Median is 6 |
| Decide which data item has occurred most often | Mode:<br>Mode is 5 |

## Worked example Y.2

Find the range and the interquartile range of the following numbers.

1, 12, 9, 9, 15, 7, 5

Range = 15 − 1
= 14

Order the data then split into two halves

Data in order:
1,5,7,9,9,12,15
$LQ = 5$, $UQ = 12$, so $IQR = 7$

## Worked example Y.3

The table shows the masses of eggs recorded in a cumulative frequency table.

(These are the data from Worked example X.2.)

| Mass of eggs, in g | Frequency |
|---|---|
| [100,120[ | 26 |
| [120,140[ | 52 |
| [140,160[ | 84 |
| [160,180[ | 60 |
| [180,200[ | 12 |

Draw a cumulative frequency graph and use it to answer the following questions:

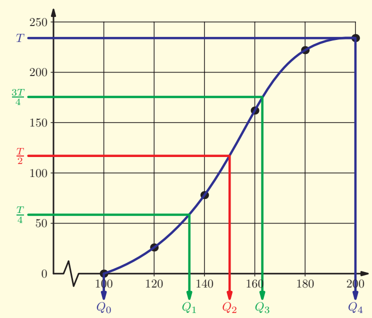
(a) Find the median and interquartile range of the eggs data.

(b) Find the 90th percentile of the mass of eggs.

(c) The top 10% of eggs are classed as extra large. What range of masses are extra large?

The total frequency is 234, so draw lines across from the *y*-axis at $0.5 \times 234 = 117$ for the median, and at $0.75 \times 234 = 176.5$ and $0.25 \times 234 = 59.5$ for the upper and lower quartiles. Where these horizontal lines meet the cumulative frequency curve, draw downward to the *x*-axis to find the values of the median and quartiles

(a)



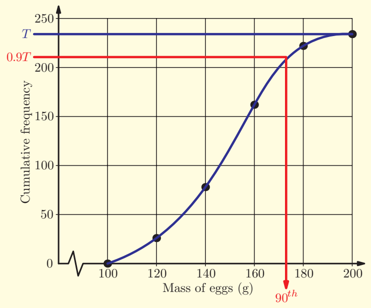

Read off the values on the *x*-axis shown by the constructed lines

Median ($Q_2$) = 150 g
Upper Quartile ($Q_3$) = 163
Lower Quartile ($Q_1$) = 134
IQR = $Q_3 - Q_1$ = 29 g

continued . . .

The total frequency is $T = 234$, so draw a line across from the $y$-axis at 90% of $T$, which is $0.9 \times 234 \approx 211$

(b)




Read off the values on the $x$-axis shown by the constructed line

From the diagram the 90th percentile is approximately 173 g

Top 10% is all the data above the 90th percentile

(c) Eggs with masses in the range [173,200[ are classified as extra large

## Exercise Y

**1.** Find (i) the mean (ii) the median (iii) the mode of the following data sets:

(a) 3,6,4,4,4

(b) 8,12,6,9,9,10

(c) 2,2,4,4,6,12

(d) −2,0,7,9,11,15,20

(e) 98,100,107,111,115,120,109

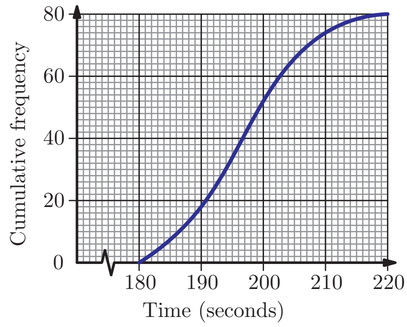
(f) 0.2,0.2,0.4,0.4,0.6,1.2

**2.** For each set of data below calculate the range and interquartile range:

(a) (i) 19.0,23.4,36.2,18.7,15.7 (ii) 0.4,−1.3,7.9,8.4,−9.4

(b) (i) 28,31,54,28,17,30 (ii) 60,18,42,113,95,23

**3.** 80 students were asked to solve a simple word puzzle and their times, in seconds, were recorded. The results are shown on the following cumulative frequency graph.




Estimate:

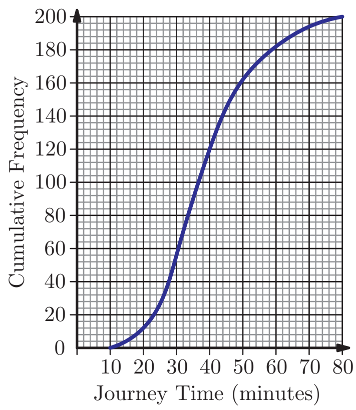
(a) the median

(b) the interquartile range.

The middle 50% of students took between *c* and *d* seconds to solve the puzzle.

(c) Write down the values of *c* and *d*.

**4.** The cumulative frequency curve below indicates the amount of time 200 students spend travelling to school.




(a) Estimate the percentage of students who spend between 30 and 50 minutes travelling to school.

(b) If 80% of the students spend more than *x* minutes travelling to school, estimate the value of *x*.

# Section Z Quadratics

## Completing the square

A quadratic expression $ax^2+bx+c$ can be written in the form $a(x-h)^2+k$. This is called the completed square form. To find it, it is best to split the process into three parts:

1. Take out a factor of $a$.
2. Complete the square of the remaining function.
3. Multiply through by $a$.

This is because it is relatively simple to complete the square when $a=1$.

We can use the following key point:

KEY POINT Z.1

$$x^2+bx+c=\left(x+\frac{b}{2}\right)^2-\frac{b^2}{4}+c$$

**Worked example Z.1**

Write $2x^2-2x-4$ in the completed square form.

Take out the factor of $a$

$$2x^2-2x-4$$
$$=2\left[x^2-x-2\right]$$

Use Key point Z.1

$$=2\left[\left(x-\frac{1}{2}\right)^2-\frac{1}{4}-2\right]$$
$$=2\left[\left(x-\frac{1}{2}\right)^2-\frac{9}{4}\right]$$

Finally, multiply out the first bracket to get the completed square form

$$=2\left(x-\frac{1}{2}\right)^2-\frac{9}{2}$$

## Solving quadratic equations using the quadratic formula

KEY POINT Z.2

The solutions of the equation $ax^2+bx+c=0$ are:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

**EXAMINERS' HINT**

This formula is given in the Formula booklet (under 'Solutions of a quadratic equation').

### Worked example Z.2

Use the quadratic formula to find the solution of $x^2 - 5x - 3 = 0$.

It is not obvious how to factorise the quadratic expression, so use the quadratic formula

$$x = \frac{5 \pm \sqrt{(-5)^2 - 4 \times 1 \times (-3)}}{2}$$

$$= \frac{5 \pm \sqrt{37}}{2}$$

The roots are

$$\frac{5+\sqrt{37}}{2} = 5.54 \text{ and } \frac{5-\sqrt{37}}{2} = -0.541 \text{ (3SF)}$$

**EXAM HINT**

In International Baccalaureate® exams you should either give exact answers $\left(\text{such as } \frac{5+\sqrt{37}}{2}\right)$ or round your answers to 3 significant figures, unless explicitly told otherwise. See Prior learning section B for rules of rounding.

## Solving quadratic inequalities

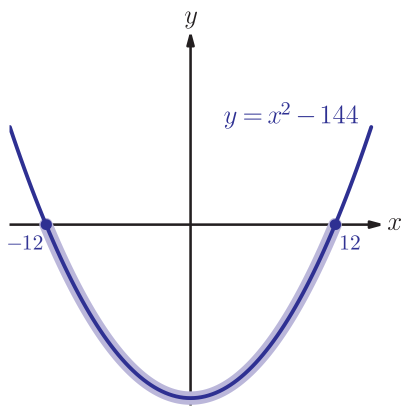
An inequality involving a square term is called a quadratic inequality. For example:

$$x^2 - 144 < 0$$

Quadratic inequalities cannot be solved in the same way you solve a linear inequality.

KEY POINT Z.3

To solve quadratic inequalities always sketch the graph.




We are looking for the values of $x$ which give a negative value of $y$; in other words, for which the graph is below the $x$-axis. The diagram shows that such values of $x$ lie between the two zeros of the function.

$$x^2 - 144 = 0 \text{ when } x = \pm 12$$

$$\therefore\ -12 < x < 12$$

**EXAM HINT**

The inequality $-12 < x < 12$ can be written using interval notation as $x \in \,]-12,12[$. See Sections G and I of the Prior learning.

## Worked example Z.3

Solve the inequality $x^2 - 6x > 7$.

Rearrange so that the RHS (right-hand side) is zero

$x^2 - 6x > 7$
$x^2 - 6x - 7 > 0$

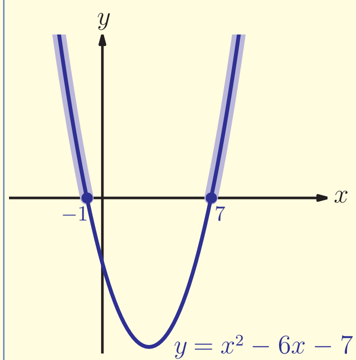
We are looking for the values of $x$ for which the graph of $y = x^2 - 6x - 7$ is above the $x$-axis. So we need to find the solutions to $x^2 - 6x - 7 = 0$ first. We can do this using the quadratic formula

Solve $x^2 - 6x - 7 = 0$:

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4 \times 1 \times (-7)}}{2}$$

$= 7$ or $-1$

We can now sketch the graph and highlight the desired part




There are two parts of the graph which give the required values of $x$, so we need to write two inequalities

$\therefore x < -1$ or $x > 7$

**EXAM HINT**

Note that we write $x < -1$ *or* $x > 7$ because there are two separate intervals for $x$. Using interval notation, this can be written as $x \in \,]-\infty, -1[\, \cup \,]7, \infty[$.

## Exercise Z

**1.** Write the following expressions in the form $a(x-k)^2 + h$:

(a) (i) $x^2 - 6x + 4$ (ii) $x^2 - 10x + 21$

(b) (i) $x^2 + 4x + 1$ (ii) $x^2 + 6x - 3$

(c) (i) $2x^2-12x+5$ (ii) $3x^2+6x+10$

(d) (i) $-x^2+2x-5$ (ii) $-x^2-4x+1$

(e) (i) $x^2+3x+1$ (ii) $x^2-5x+10$

(f) (i) $2x^2+6x+15$ (ii) $2x^2-5x-1$

**2.** Find the solutions of the following equations:

(a) (i) $x^2-3x+1=0$ (ii) $x^2-x-1=0$

(b) (i) $3x^2+x-2=0$ (ii) $2x^2-6x+1=0$

(c) (i) $4+x-3x^2=0$ (ii) $1-x-2x^2=0$

(d) (i) $x^2-3=4x$ (ii) $3-x=2x^2$

**3.** Solve the following quadratic inequalities:

(a) (i) $x^2\le 8$ (ii) $x^2<5$

(b) (i) $x^2>6$ (ii) $x^2\ge 12$

(c) (i) $(x-4)(x+1)>0$ (ii) $(2x-5)(3x+2)<0$

(d) (i) $(3-x)(x+1)<0$ (ii) $(4-x)(x-2)>0$

(e) (i) $(3-x)(12-x)>0$ (ii) $(2-x)(-2-x)<0$

**4.** Solve the following inequalities:

(a) (i) $x^2-5x+6<0$ (ii) $x^2+x-6<0$

(b) (i) $x^2-4x-12\ge 0$ (ii) $x^2+7x+6\ge 0$

(c) (i) $2x^2+x>6$ (ii) $3x^2-x>10$

(d) (i) $2x^2+3x-5\le 0$ (ii) $5x^2+6x+1\le 0$

# Prior learning: worksheet answers

## Exercise A

1. (a) $-2$ (b) $-16$
(c) $-25$ (d) $25$
(e) $111$ (f) $3$

2. (a) $1\frac{1}{6}$ (b) $2\frac{3}{35}$
(c) $\frac{7}{46}$ (d) $-\frac{2}{7}$
(e) $22\frac{7}{15}$ (f) $2\frac{21}{44}$

3. (a) $\frac{7}{10}$ (b) $\frac{3}{4}$
(c) $\frac{7}{5}$ (d) $\frac{46}{5}$
(e) $1$ (f) $\frac{1}{10}$

4. (a) $3\frac{7}{9}$ (b) $1\frac{2}{3}$
(c) $3\frac{1}{2}$ (d) $1\frac{2}{9}$
(e) $1\frac{19}{28}$ (f) $1\frac{3}{11}$

## Exercise B

1. (a) 34600 (b) 23.54
(c) 0 (d) 100000
(e) 789.6 (f) 13.0

2. (a) $1.598\times10^{9}$
(b) $5.64\times10^{-5}$
(c) $2.356\times10^{0}$
(d) $2.356\times10^{1}$
(e) $2.356\times10^{-1}$
(f) $1.89765\times10^{7}$

3. (a) 65000000
(b) 0.00712
(c) 85.4
(d) 8.88
(e) 0.0000001152
(f) 236540000

## Exercise C

1. (a) False (b) False
(c) True (d) True
(e) False (f) False
(g) True (h) False

2. (a) 32 (b) 49
(c) 216 (d) 27
(e) 17 (f) 25
(g) 18 (h) 40
(i) $\frac{8}{27}$ (j) $\frac{1}{16}$
(k) $-\frac{125}{27}$

3. (a) $\frac{1}{3}$ (b) $\frac{1}{6}$
(c) $\frac{1}{49}$ (d) $\frac{1}{64}$
(e) $\frac{3}{4}$ (f) $\frac{6}{5}$

4. (a) 2 (b) 3
(c) 8 (d) 1
(e) $\frac{1}{5}$

## Exercise D

1. (a) $5\sqrt{5}$ (b) $7\sqrt{5}$
(c) $5\sqrt{5}$ (d) $-5\sqrt{5}$
(e) $2\sqrt{5}$ (f) $11\sqrt{5}$

2. (a) $\sqrt{32}$ (b) $\sqrt{147}$
(c) $\sqrt{63}$ (d) $\sqrt{108}$
(e) $\sqrt{18}$ (f) $\sqrt{72}$

3. (a) $3+\sqrt{3}$ (b) $2\sqrt{3}$
(c) $5+3\sqrt{3}$ (d) $-5+\sqrt{3}$
(e) $28-16\sqrt{3}$ (f) $20-10\sqrt{3}$

4. (a) $\frac{\sqrt{7}}{7}$ (b) $\frac{\sqrt{6}-2}{2}$
(c) $\frac{12+3\sqrt{3}}{13}$ (d) $\sqrt{2}+1$
(e) $\frac{\sqrt{7}-\sqrt{5}+\sqrt{35}-1}{6}$
(f) $\frac{11+6\sqrt{2}}{7}$

5. (a) $\frac{3}{2}\sqrt{2}$ (b) $5-\sqrt{2}$
(c) $2\sqrt{2}$

## Exercise E

1. (a) $2^3\times3^2$ (b) $2\times5^2$
(c) $2^5$ (d) $3^3$
(e) 29 (f) $7\times11\times13$

2. (a) $\gcd(12,25)=1$, $\text{lcm}(12,25)=300$
(b) $\gcd(17,16)=1$, $\text{lcm}(17,16)=272$
(c) $\gcd(18,27)=9$, $\text{lcm}(18,27)=54$
(d) $\gcd(42,56)=14$, $\text{lcm}(42,56)=168$
(e) $\gcd(26,39)=13$, $\text{lcm}(26,39)=78$
(f) $\gcd(14,32)=2$, $\text{lcm}(14,32)=224$
(g) $\gcd(50,25)=25$, $\text{lcm}(50,25)=50$

## Exercise F

1. (a) 40 : 60
(b) 544 : 256
(c) 112.5 : 87.5
(d) 70 : 140 : 280
(e) 2.5 :17.5 : 40
(f) 3.8 : 7.6 : 7.6

2. (a) $a=\frac{b}{2}$ (b) $r=\frac{4s}{3}$
(c) $x=4.8$ (d) $y=17.5$
(e) $p=\frac{q^2}{20}$ (f) $r=2$

## Exercise G

1. (a) $\{5,15\}$
(b) $\{10\}$
(c) $\{2,3,5,7,11,13,15,17,19\}$
(d) $\{1,9,15\}$
(e) 2
(f) 11

2. (b) $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$
(c) $\mathbb{Q}$, $\mathbb{R}$
(d) $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$
(e) $\mathbb{Q}$, $\mathbb{R}$
(f) $\mathbb{Q}$, $\mathbb{R}$
(g) $\mathbb{R}$
(h) $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$
(i) $\mathbb{R}$

## Exercise H

1. (a) (i) 0.667 (ii) 0.15
(b) (i) 0.571 (ii) 0.4
(c) (i) 0.429 (ii) 0.1

**2.** (a) 5 (b) 75

**3.** (a) (i) $\frac{1}{2}$ (ii) $\frac{1}{4}$

(b) (i) $\frac{1}{13}$ (ii) $\frac{4}{13}$

(c) (i) $\frac{91}{26}$ (ii) $\frac{1}{3}$

**4.** (a) (i) $\frac{21}{51}$ (ii) $\frac{1}{3}$

(b) (i) $\frac{11}{15}$ (ii) $\frac{3}{5}$

(c) (i) 0 (ii) 1

## Exercise I

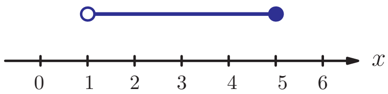
**1** (a) 6 (b) 14

(c) 8 (d) 16

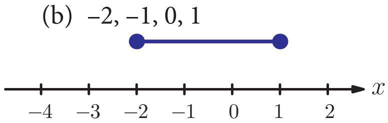
(e) 0.6 (f) $\frac{1}{5}$

(g) 3 (h) $\frac{1}{5}$

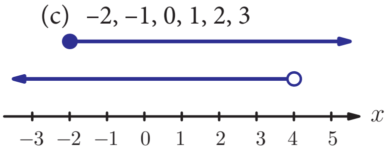
**2.** (a) 2, 3, 4, 5



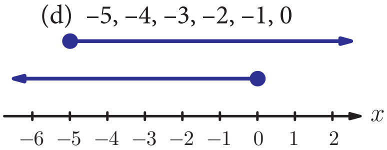

(b) −2, −1, 0, 1



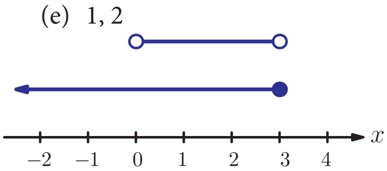

(c) −2, −1, 0, 1, 2, 3



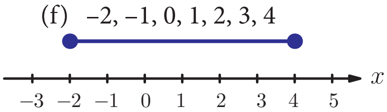

(d) −5, −4, −3, −2, −1, 0



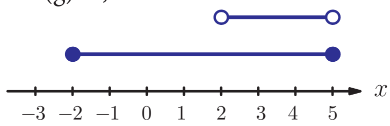

(e) 1, 2



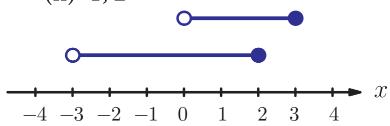

(f) −2, −1, 0, 1, 2, 3, 4




(g) 3, 4




(h) 1, 2




**3.** (a) $1 \le x \le 2$ (b) $x \le 4$

(c) $1 \le x < 6$ (d) $x > -3$

(e) $-3 < x < 3$ (f) $x \ge 5$

**4.** (a) $x \in ]2,3]$

(b) $x \in [1.5[\cup[7,\infty[$

(c) $x \in ]-\infty,-2]\cup[2,\infty[$

(d) $x \in ]-\infty,-1]\cap]1,3[=\varnothing$

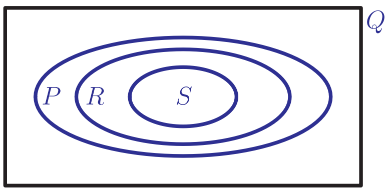
**5.** (a) $|x| < 2$ (b) $|x| \le 5$

(c) $|x| \le 4$ (d) $|x| > 2$

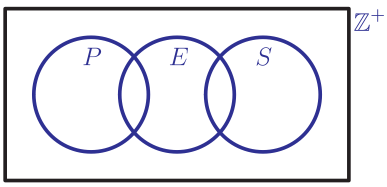
(e) $|x| \ge 10$

## Exercise J

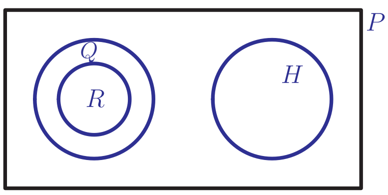
**1.** (a)



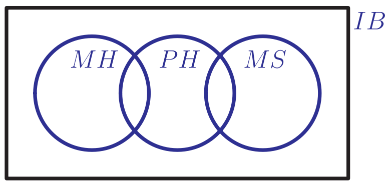

(b)



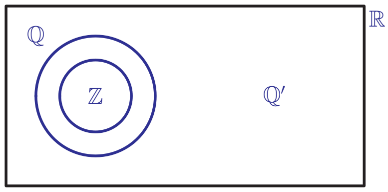

(c)



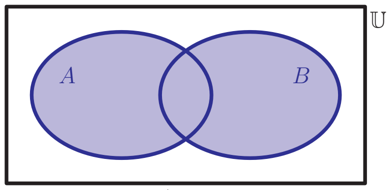

(d)




(e)

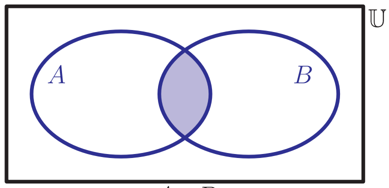



**2.** (a)



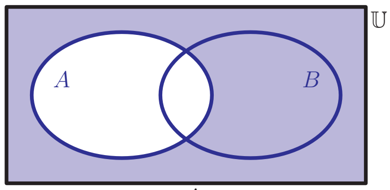

$A \cup B$

(b)



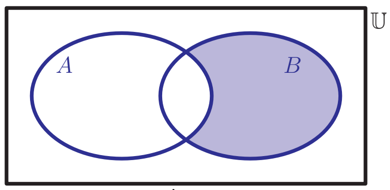

$A \cap B$

(c)



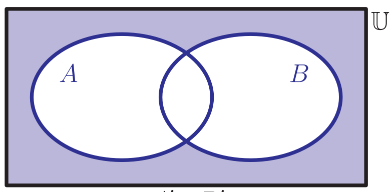

$A'$

(d)



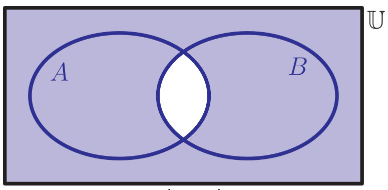

$A' \cap B$

(e)




$A' \cap B'$

(f)




$A' \cup B'$

## Exercise K

**1.** (a) $2+x+y$
(b) $2$
(c) $5x-2xy+x^2-y^2$
(d) $11z^2$
(e) $x^2+2$
(f) $x^2+3xy+3xz+5yz$

**2.** (a) $x^2+8x+15$
(b) $z^2-1$
(c) $6a^2-a-2$
(d) $xy+x+y+1$
(e) $x^2-6x+9$
(f) $4x^2+4xy+y^2$
(g) $a^3-1$
(h) $x^3+6x^2+11x+6$
(i) $x^2+2xy+y^2-2x-2y+1$

**3.** (a) False (b) True
(c) True (d) False
(e) True (f) False
(g) False (h) True
(i) False (j) False

## Exercise L

**1.** (a) $x=-\frac{3}{4}$ (b) $x=\frac{1}{3}$
(c) $x=\frac{10}{3}$ (d) $x=-\frac{7}{2}$
(e) $x=\frac{4}{7}$ (f) $x=1$

**2.** (a) $x>\frac{9}{2}$ (b) $x\le\frac{1}{3}$
(c) $x>2$ (d) $x<\frac{4}{5}$

## Exercise M

**1.** (a) $A=3\frac{1}{2}$
(b) $B=36$
(c) $C=3\sqrt{5}$

**2.** (a) $x=\frac{A-7}{3}$
(b) $x=\sqrt{3-B}$
(c) $x=\frac{1}{C-1}$
(d) $x=\frac{1-(D-7)^2}{3}$
(e) $x=\frac{3+2E}{E-1}$
(f) $x=\frac{10-3F}{2F-1}$
(g) $x=\sqrt{\frac{1-2G}{G-1}}$
(h) $x=\frac{a-8H}{H-1}$
(i) $x=\frac{a-cI}{dI-b}$

**3.** (a) $z=15x+5$
(b) $z=12x+5$
(c) $z=18x^2$
(d) $z=x^2+4x+8$
(e) $z=3x+3$
(f) $z=\frac{x}{2-x}$

## Exercise N

**1.** (a) $4(2x+3y)$
(b) $5y(1-2y)$
(c) $2x(9x^2y-2)$
(d) $7ab(2ab-1)$
(e) $2xy(16x^2+2xy+y^3)$
(f) $3a(5ab^2-4b+3a)$

**2.** (a) $(x-1)(x+3)$
(b) $(x-7)(x+5)$
(c) $(a-10)(a+2)$
(d) $(b-9)(b+9)$
(e) $2(x+2)(x+3)$
(f) $5(x-3)(x+3)$

**3.** (a) $(x+1)(y+1)$
(b) $(x+1)(x+y)$
(c) $(a+2)(b+3)$
(d) $(2a+1)(3b-4)$
(e) $(p-1)(q-1)$
(f) $(2pq-5)(3p-2)$

## Exercise O

**1.** (a) $\frac{1}{x}$
(b) $x+2$
(c) $x$
(d) No simplification
(e) $\frac{x+1}{x+2}$
(f) $\frac{a-3}{a+3}$

**2.** (a) $\frac{21x+5}{30}$
(b) $\frac{x^2+4}{2x}$
(c) $\frac{2x+3}{(x+2)(x+1)}$
(d) $\frac{x^2+2x+3}{x(x+1)}$
(e) $\frac{7x^2-2x+3}{(x+2)(2x-1)}$
(f) $\frac{20x+2}{9x^2-1}$

**3.** (a) $\frac{3x}{5}$
(b) $4$
(c) $\frac{1}{(x+5)(x-2)}$
(d) $\frac{2x+7}{(x-2)(x-1)}$
(e) $\frac{3}{2(x-4)}$
(f) $6$

## Exercise P

**1.** (a) $\frac{3}{2}$
(b) $\frac{4}{5}$
(c) $0$

**2.** (a) $\left(2, 5\frac{1}{2}\right)$
(b) $\left(6\frac{1}{2}, 0\right)$
(c) $(1, -3)$

**3.** (a) $\sqrt{13}$
(b) $\sqrt{41}$
(c) $8$

## Exercise Q

**1.** (a) $(2,7)$ (b) $\left(2\frac{1}{2}, -\frac{1}{2}\right)$
(c) $\left(\frac{5}{4}, -\frac{3}{4}\right)$ (d) $(1,1)$
(e) $(2,3)$ (f) $(2,3)$

**2.**
(a) $(x,y)=(3,4)$ or $(4,3)$
(b) $(x,y)=(4,14)$ or $(-6,4)$
(c) $(x,y)=(4,3)$ or $\left(17\frac{1}{2},-6\right)$
(d) $(x,y)=(5,4)$ or $\left(8,2\frac{1}{2}\right)$

## Exercise R

**1.** (a) $2,(0,-7)$ (b) $-1,(0,5)$
(c) $-3,(0,9)$ (d) $3,(0,-8)$
(e) $2,\left(0,\frac{1}{5}\right)$ (f) $-\frac{2}{3},(0,0)$

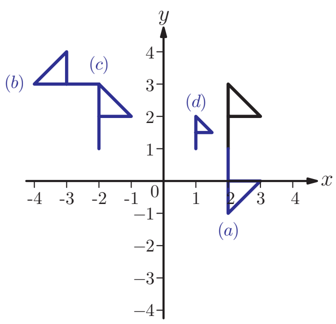
**2.** (a) $7x-y=4$
(b) $x+2y=6$
(c) $5x+y=0$
(d) $x+y=6$
(e) $y=6$
(f) $x=4$

**3.** (a) $y=3x-2$
(b) $y=-x+6$
(c) $y=\frac{3}{2}x$
(d) $y=-x+6$
(e) $y=2x-12$
(f) $y=-\frac{5}{3}x+6$

## Exercise S

**1.** (a) Reflection in $x=-1$
(b) Rotation $180°$ about $(0,0)$
(c) Enlargement s.f.2 centred at $(0,0)$
(d) Translation $\begin{pmatrix}-4\\-3\end{pmatrix}$
(e) Reflection in $y=x$
(f) Rotation $90°$ anticlockwise about $(0,0)$

**2.**




## Exercise T

**1.** (a) 3.2 (b) 1.2
(c) 7.5

**2.** (a) Yes: SSS (b) Yes: SAS
(c) No (d) Yes: ASA

## Exercise U

**1.** $a=5$, $b=5.44$,
$c=2.54$, $d=2.91$,
$e=4.67$, $f=5.01$

**2.** $A=41.8°$, $B=35.5°$,
$C=54.5°$, $D=65.4°$,
$E=26.6°$, $F=30°$

**3.** 051°

## Exercise V

**1.** (a) 6 (b) 9
(c) 7.5 (d) $16\pi=50.3$

**2.** (a) $24\pi=75.4$
(b) 100
(c) $36\pi=113$
(d) $24\pi=75.4$

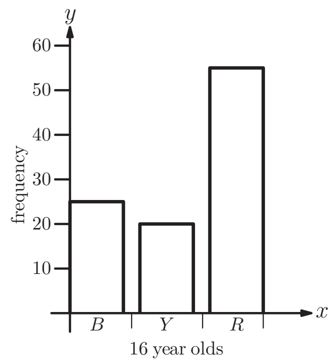
## Exercise W

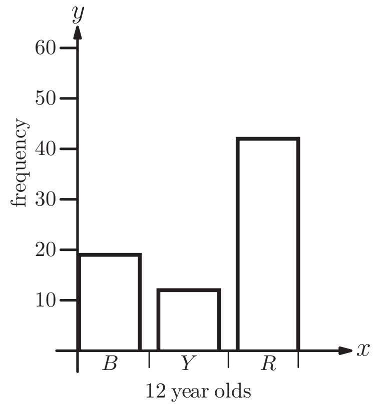
**1.** (a) 20°: Tangent meets radius at right-angle
(b) 50°: Angle at centre is double angle at circumference
(c) 80°: Opposite angles in a cyclic quadrilateral sum to 180°
(d) 30°: Angles in the same segment are equal

**2.** 80° and 100°

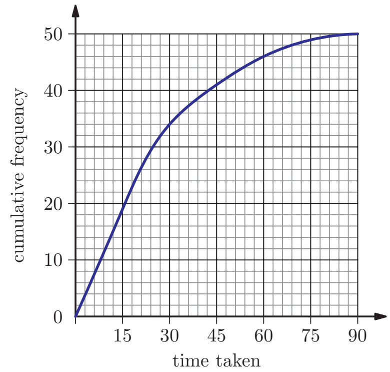
## Exercise X

**1.** (a) (i) 20 (ii) 25 (iii) 55
(b) (i) 90° (ii) 60° (iii) 210°
(c)



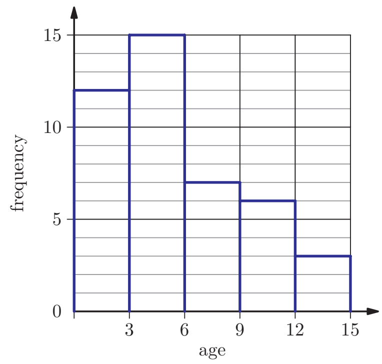




**2.** (a)




(b) About 39

**3.** (a)




(b) About 31

**4.** A3, B2, C1

## Exercise Y

**1.** (a) (i) 4.2
(ii) 4 (iii) 4
(b) (i) 9 (ii) 9
(iii) 9
(c) (i) 5 (ii) 4
(iii) 2,4
(d) (i) 8.57 (ii) 9
(iii) None
(e) (i) 110 (ii) 109
(iii) None
(f) (i) 0.5 (ii) 0.4
(iii) 0.2,0.4

2. (a) (i) range $= 20.5$, IQR $= 12.6$
(ii) range $= 17.8$, IQR $= 13.5$
(b) (i) range $= 37$, IQR $= 3$
(ii) range $= 95$, IQR $= 72$

3. (a) 197
(b) 12
(c) $c = 191, d = 203$

4. (a) 53%
(b) 28 minutes

## Exercise Z

1. (a) (i) $(x-3)^2 - 5$
(ii) $(x-5)^2 - 4$
(b) (i) $(x+2)^2 - 3$
(ii) $(x+3)^2 - 12$
(c) (i) $2(x-3)^2 - 13$
(ii) $3(x+1)^2 + 7$
(d) (i) $-(x-1)^2 - 4$
(ii) $-(x+2)^2 + 5$
(e) (i) $(x+1.5)^2 - 1.25$
(ii) $(x-2.5)^2 + 3.75$
(f) (i) $2(x+1.5)^2 + 10.5$
(ii) $2(x-1.25)^2 - 4.125$

2. (a) (i) $x = \frac{3 \pm \sqrt{5}}{2}$
(ii) $x = \frac{1 \pm \sqrt{5}}{2}$
(b) (i) $x = -1, \frac{2}{3}$
(ii) $x = \frac{3 \pm \sqrt{7}}{2}$
(c) (i) $x = \frac{4}{3}, -1$
(ii) $x = \frac{1}{2}, -1$
(d) (i) $x = 2 \pm \sqrt{7}$
(ii) $x = 1, -\frac{3}{2}$

3. (a) (i) $-2\sqrt{2} \le x \le 2\sqrt{2}$
(ii) $-\sqrt{5} < x < \sqrt{5}$
(b) (i) $x < -\sqrt{6}$ or $x > \sqrt{6}$
(ii) $x \le -2\sqrt{3}$ or $x \ge 2\sqrt{3}$
(c) (i) $x < -1$ or $x > 4$
(ii) $-\frac{2}{3} < x < \frac{5}{2}$
(d) (i) $x < -1$ or $x > 3$
(ii) $2 < x < 4$
(e) (i) $x < 3$ or $x > 12$
(ii) $-2 < x < 2$

4. (a) (i) $2 < x < 3$
(ii) $-3 < x < 2$
(b) (i) $x \le -2$ or $x \ge 6$
(ii) $x \le -6$ or $x \ge -1$
(c) (i) $x < -2$ or $x > 1.5$
(ii) $x < -5/3$ or $x > 2$
(d) (i) $-2.5 \le x \le 1$
(ii) $-1 \le x \le -0.2$